NAME:

Purpose

To change ratios and fractions to percents

Math Words

ratio	Percent is a ratio that compares a number to 100.
compared to	You can write the ratio 3 compared to 100 several ways, such as: $3{:}100 \quad \dfrac{3}{100} \quad 0.03 \quad 3\%$
per hundred	32% means 32 for every hundred, or 32 per hundred.

Starter Problem

24 out of 60 kids surveyed have attended a professional sporting event. Write the percent for this ratio.

$$\frac{24}{60} = \underline{\hspace{2cm}}\%$$

246   Math Pathways & Pitfalls® ✳ Unit 6 ✳ Lesson 5

NAME:

Starter Problem

24 out of 60 kids surveyed have attended a professional sporting event.
Write the percent for this ratio.

$$\frac{24}{60} = \underline{\hspace{2cm}} \%$$

Student Thinking

I could divide 60 into 24 to get a decimal, but it's easier to simplify first. $\frac{24}{60}$ is equal to $\frac{2}{5}$. Then I divided 2 by 5 to get 0.40 or 40%.
It makes sense because 24 out of 60 is a little less than half of the kids.

Abby

OK

$$\frac{24}{60} = \frac{2}{5} = 0.40 = 40\%$$

I divided 24 into 60 and got 25%.

Walt

Pitfall

$$24\overline{)60.0} \quad \frac{24}{60} = 25\%$$
$$\begin{array}{r} 25 \\ 48 \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

Things to Remember

* ....

 ....

* ....

 ....

STOP

NAME:

Our Turn

Write the percent for each ratio.

1. $\dfrac{6}{40}$ = _____ %

2. $\dfrac{8}{5}$ = _____ %

3. Carlos paid a $2 shipping charge for a package of art supplies that cost $10. What percent of the total costs did he pay for shipping? (Round your answer to the nearest tenth of a percent.)

$\dfrac{2}{12}$ ≈ _____ %

NAME:

My Turn

Write the percent for each ratio.

1. $\dfrac{3}{8}$ = _____ %

2. $\dfrac{80}{40}$ = _____ %

3. A school paper reported that 10 out of every 15 students wore a school T-shirt on Friday. What percent is this? (Round your answer to the nearest tenth of a percent.)

$\dfrac{10}{15}$ ≈ _____ %

NAME:

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. $\dfrac{17}{20} =$ _____

   ○ 3%   ○ 8.5%   ○ 11.8%   ○ 85%

2. $\dfrac{5}{6} \approx$ _____

   ○ 0.83%   ○ 1.2%   ○ 83.3%   ○ 56%

✂ -

250   Math Pathways & Pitfalls® ✳ Unit 6 ✳ Lesson 5

NAME:

Writing Task Mini Lesson

Explain how you know $\frac{21}{24}$ equals 87.5%.

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Changing Ratios to Percents

Lesson at a Glance

Prior Learning Needed

- Find equivalencies among fractions, whole numbers, mixed numbers, and decimals

- Use a division algorithm to find a decimal quotient

Lesson Preparation

- Study Lesson Foundation

- Review Teaching Guide and Student Pages

- Prepare stapled packet of Student Pages 1–4 for each student

- Copy and cut in half Student Pages 5 and 6

- Post *Discussion Builders* poster

Mathematical goals

✳ Find percents for ratios by first finding an equivalent decimal

✳ Round to the nearest tenth percent

Mathematical language and reasoning goals

✳ Compare a ratio to common benchmarks such as 1 (or 100%) and 1/2 (or 50%)

✳ Use reasoning to know when to simplify a ratio before changing it to a decimal or percent

LESSON ROADMAP			MATERIALS
CORE LESSON: DAY 1	GROUPING	TIME	❍ *Discussion Builders* poster
Opener			❍ Projector (optional)
Discussion Builders			❍ Student Pages 1 and 2
Purpose	👥	🕐	❍ Teaching Guide
Math Words			❍ Calculators for checking division (suggested)
Starter Problem	👤	🕐	
Discussion			
Student Thinking	👤 👥 👥	🕐	
Things to Remember	👥	🕐	
Reflection			
CORE LESSON: DAY 2			❍ Clipboard Prompts, page 37
Review and Practice			❍ Student Page 2 (completed day 1)
Review Day 1 Lesson	👥	🕐	❍ Student Pages 3 and 4
Our Turn	👥 👥	🕐	❍ Teaching Guide
My Turn	👤	🕐	❍ Calculators for checking division (suggested)
MINI LESSONS: 2–3 DAYS LATER			❍ Student Pages 5 and 6
Assess and Reinforce			❍ Teaching Guide
Multiple Choice Mini Lesson	👤 👥 👥	🕐	❍ Calculators for checking division (suggested)
Writing Task Mini Lesson	👤 👥	🕐	

Lesson Foundation

Starter Problem

24 out of 60 kids surveyed have attended a professional sporting event. Write the percent for this ratio.

$$\frac{24}{60} = \underline{\hspace{2cm}} \%$$

Student Thinking

Abby

I could divide 60 into 24 to get a decimal, but it's easier to simplify first. $\frac{24}{60}$ is equal to $\frac{2}{5}$. Then I divided 2 by 5 to get 0.40 or 40%. It makes sense because 24 out of 60 is a little less than half of the kids.

OK

$$\frac{24}{60} = \frac{2}{5} = 0.40 = 40\%$$

Walt

I divided 24 into 60 and got 25%.

Pitfall

$$24\overline{)60.0} \quad \frac{25}{} \quad \frac{48}{120} \quad \frac{120}{0}$$

$$\frac{24}{60} = 25\%$$

MATHEMATICAL INSIGHTS & TEACHING TIPS

Finding Percents for Ratios

Percents, decimals, and fractions are all ways of writing a ratio. To write a ratio that is in fraction form as a percent, first change the ratio to a decimal in hundredths. Abby realized that since the fraction line separating the numerator and denominator means "divided by," she could divide to find a decimal equivalent for 24/60. However, she simplified the ratio first to find that 24/60 is equivalent to 2/5 or 40%. Students could be asked to prove that 1/5 is equivalent to 20% and 2/5 is equivalent to 40% by drawing a picture as they did in earlier lessons.

Pitfall Walt mistakenly divided the lesser number into the greater number instead of the denominator into the numerator. He also made a mistake with the decimal point in his quotient.

ⓘ▸ *Encourage students to rewrite a ratio in lowest terms before dividing to find an equivalent decimal.*

MATHEMATICAL INSIGHTS & TEACHING TIPS (CONTINUED)

Dividing a Greater Number into a Lesser Number

One way to find the correct percent is by dividing the denominator into the numerator. Walt, like many students, may have incorrectly thought that you can't divide a greater number like 60 into a lesser number like 24, so he switched and divided 24 into 60. This gave him the incorrect answer of 25, and he simply appended a percent to it.

ⓘ▶ *Students may have been told in earlier grades that they can't divide a lesser number by a greater number. Provide practice with problems like "What percent is 8 out of 16?" Talk about why dividing 8 into 16 and getting 2% or 20% doesn't make sense. This pitfall is particularly tempting because 8 divides evenly into 16.*

Repeating Decimals and Rounding

Repeating decimals are frequently encountered when changing a ratio to a decimal. For example: 2/3 = 0.6666…, 20/45 = 0.444…, and 3/11 = 0.2727…. It is common to round the decimal to the thousandths place and give the percent to the nearest tenth of a percent (e.g., 2/3 rounds to 0.667, so 2/3 ≈ 66.7%). Interpreting ratio-percent equivalencies is complex and important. For example, stating that 5/8 equals 62.5% means that 5 for every 8 is the same as 62.5 for every 100.

MATHEMATICAL DISCUSSION SUPPORT

Ask students to write phrases such as "24 divided by 60" and "24 into 60" in symbol form. Point out that 24 ÷ 60 and 60 ÷ 24 are not equivalent statements. Also point out that you can divide a lesser number by a greater number and that the quotient is a number less than 1.

English Learner Access

Have students practice using and interpreting a variety of phrases involving ratios, such as: "3 plants for every 4 seeds is 75% of the seeds germinating," "3 out of 12 people is 25% of all the people," or "27 baskets out of every 90 shots is a success rate of 30%." Using these phrases in the context of real situations is helpful for understanding and developing mathematical language.

**Core Lesson
Day 1**

Opener

Review *Discussion Builders*

Read the poster. **Suggest** a section to focus on today:
Presenting Alternative Ideas, Expanding on Others' Ideas, or *Posing Additional Questions.*

Purpose

Distribute stapled packets of Student Pages 1–4. **Project** an image of page 1 (optional).

Call on a student to read the purpose.

Math Words

Point to and say the first math word. **Ask** students to repeat it aloud or silently.

Read the sentence containing the word.

Give an example using objects or drawings.

Repeat for the other math words.

Starter Problem

Read the Starter Problem. **Call on** a student to restate it in his/her own words.

Think about what the Starter Problem means. WAIT
Try to use what you understand to solve the problem on your own.

I'll walk around and write notes about things we need to discuss. Look out for pitfalls! WAIT

Look at your work. It's easy to have a pitfall in this type of problem. You might also have made a pitfall if your answer is less than 40%.

Don't worry. Next we'll discuss how two imaginary students solved this problem. One has a pitfall! You may keep your solution private, but bring up your ideas in the discussion.

STUDENT PAGE 1

Purpose
To change ratios and fractions to percents

Math Words

ratio	Percent is a ratio that compares a number to 100.
compared to	You can write the ratio 3 compared to 100 several ways, such as: 3:100 $\frac{3}{100}$ 0.03 3%
per hundred	32% means 32 for every hundred, or 32 per hundred.

Starter Problem

24 out of 60 kids surveyed have attended a professional sporting event. Write the percent for this ratio.

$$\frac{24}{60} = \underline{\hspace{2cm}} \%$$

Discussion

Student Thinking

STUDENT PAGE 2

Abby

I could divide 60 into 24 to get a decimal, but it's easier to simplify first. $\frac{24}{60}$ is equal to $\frac{2}{5}$. Then I divided 2 by 5 to get 0.40 or 40%. It makes sense because 24 out of 60 is a little less than half of the kids.

OK

$$\frac{24}{60} = \frac{2}{5} = 0.40 = 40\%$$

Ask students to refer to page 2. **Read** the statement marked OK.

Explain that this statement is about the same problem students worked on earlier.

We can learn a lot about the math by studying what this student did.

Read each sentence silently and look at Abby's work. Think about what they mean. **WAIT**

Now talk with a partner about what each sentence and each part of Abby's work means.

Listen in, ask questions, and observe. **Note** potential contributions for the discussion.

Who can come up and explain why Abby simplified 24/60 and how she got 2/5?

Why did Abby divide 2 by 5? What does the line separating the numerator and denominator of a fraction mean? Why do you divide 2 by 5 rather than 5 by 2?

Talk to your neighbor about what Abby meant when she said that 40% makes sense because 24 out of 60 is a little less than half of the kids. **WAIT**

How many kids are half of 60 kids? Explain why 40% makes sense.

Call on students to state things to remember about solving problems like this.

Start a Things to Remember list on the board.

MORE DAY 1

Discussion

Student Thinking, continued

STUDENT PAGE 2

Walt

I divided 24 into 60 and got 25%.

$$24\overline{)60.0}$$ with quotient 25, subtracting 48, bringing down 120, 120, remainder 0

$$\frac{24}{60} = 25\%$$

Pitfall

Read the statement marked Pitfall. **Remind** students that this is a common pitfall.

Walt made a pitfall when he divided 60 by 24 instead of 24 by 60. Talk with your neighbor about why Walt's answer doesn't make sense. WAIT

Is the percent he wrote too high or too low? Explain.

Write the following examples on the board. **Ask** students to talk with a neighbor about whether they are true statements or not. **Remind** them to look out for pitfalls.

$$\frac{15}{20} = 15 \div 20 \qquad\qquad \frac{10}{15} = 10\overline{)15}$$

Things to Remember

Call on students to **add** to the Things to Remember list on the board. **Read** the list.

Help students summarize and record two important Things to Remember.

> **Things to Remember List (sample)**
> 1. To find a percent for a ratio, divide the numerator by the denominator to get a decimal in hundredths. Then write the quotient as a percent.
> 2. To change a decimal to a percent, move the decimal two places to the right and attach a percent sign. For example, $0.40 = 40\%$ and $0.562 = 56.2\%$.

Reflection

Ask students to reflect on the discussion process using one of the sample prompts.

> **Reflection Prompts (sample)**
> • Name a *Discussion Builder* that we used today. How did it help the discussion?
> • What *Discussion Builder* could we use next time to make the discussion even better?
> • What did someone do or say today that helped you understand the math?

**Core Lesson
Day 2**

Review and Practice

Review

Ask students to review page 2 to jog their memory.

Read the statement marked OK. Call on a student to explain how the problem was solved.

Read the statement marked Pitfall. Call on a student to explain why it is incorrect.

Call on two or three students to read an item on their Things to Remember list.

Abby

I could divide 60 into 24 to get a decimal, but it's easier to simplify first. $\frac{24}{60}$ is equal to $\frac{2}{5}$. Then I divided 2 by 5 to get 0.40 or 40%. It makes sense because 24 out of 60 is a little less than half of the kids. **OK**

$$\frac{24}{60} = \frac{2}{5} = 0.40 = 40\%$$

Walt

I divided 24 into 60 and got 25%. **Pitfall**

$$24\overline{)60.0} \quad \frac{24}{60} = 25\%$$

Our Turn

Ask students to refer to page 3.

Use the procedure below and the Clipboard Prompts to discuss students' solutions. Discuss the problems one at a time.

Read the problem.

Ask students to work with a neighbor to solve it.

Discuss one or two students' solutions.

Answer Key		
	1.	15%
	2.	160%
	3.	16.7%

Our Turn

Write the percent for each ratio.

1. $\frac{6}{40}$ = _____ %

2. $\frac{8}{5}$ = _____ %

3. Carlos paid a $2 shipping charge for a package of art supplies that cost $10. What percent of the total costs did he pay for shipping? (Round your answer to the nearest tenth of a percent.)

$\frac{2}{12}$ ≈ _____ %

My Turn

Ask students to solve the problems on page 4. Remind them to watch out for pitfalls!

After allowing time to work, read the answers. Have students use pens to mark and revise their papers.

Answer Key		
	1.	37.5%
	2.	200%
	3.	66.7%

My Turn

Write the percent for each ratio.

1. $\frac{3}{8}$ = _____ %

2. $\frac{80}{40}$ = _____ %

3. A school paper reported that 10 out of every 15 students wore a school T-shirt on Friday. What percent is this? (Round your answer to the nearest tenth of a percent.)

$\frac{10}{15}$ ≈ _____ %

Mini Lessons
(2–3 Days Later)

Assess and Reinforce

Multiple Choice Mini Lesson

Distribute Student Page 5.

Problem 1

Please read problem 1.

Talk with your neighbor about which choices don't make sense. WAIT

How do you know the correct choice must be greater than 50%

Who can show why the correct choice is 85%?

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. $\frac{17}{20}$ = _____

○ 3%   ○ 8.5%   ○ 11.8%   ● 85%

2. $\frac{5}{6} \approx$ _____

○ 0.83%   ○ 1.2%   ● 83.3%   ○ 56%

ⓘ▶ *Watch for students who incorrectly divide the denominator by the numerator to get a decimal and choose 11.8%.*

Problem 2

Read the problem and find the correct choice. WAIT

Which response is correct? Explain why.

Who can show us a way to prove the correct response is 83.3%?

Writing Task Mini Lesson

Distribute Student Page 6.

Ask a student to read the task. **Call on** students to respond with their ideas.

Jot the ideas on the board.

Write an explanation together using their ideas. **Read** it aloud.

Ask students to write an explanation on their page.

Writing Task Mini Lesson

Explain how you know $\frac{21}{24}$ equals 87.5%.

Sample Explanation: Divide 24 into 21 and you get the decimal 0.875. Move the decimal to the right two places and write a percent sign. It makes sense because 21/24 is getting close to 1 and 87.5% is also fairly close to 100%.

English Learner Access

Mathematical Discussion Support

Ask students to use drawings or show their calculations to help support their explanations. Have students read numbers such as 0.875 and 87.5% orally and explain what they mean. Prompt them with questions such as "Why does it make sense to get less than 1 for a quotient?"