Solve for x in Proportions

NAME:

Math PATHWAYS & PITFALLS

Purpose

To use scale factors to solve for x in proportions

Math Words

proportion

Two equal ratios such as 8:6 and 4:3 form a proportion. We say 8 is to 6 as 4 is to 3.

$8:6 = 4:3 \qquad \frac{8}{6} = \frac{4}{3}$

equation

You can solve a proportion equation to find an unknown part.

$$\frac{5 \text{ cans}}{7 \text{ days}} = \frac{15 \text{ cans}}{x \text{ days}}$$

scale down

To scale down 30 by a factor of 5, divide 30 by 5.

Starter Problem

Find the missing number to make the ratios equal.

$$3:x = 24:16$$

NAME:

Starter Problem

Find the missing number to make the ratios equal.

$$3:x = 24:16$$

Student Thinking

Amina

I rewrote the proportion in fraction form. The scale factor is 8, since 3 times 8 equals 24. Since *x* times 8 has to equal 16, *x* must equal 2. It checks since $\frac{3}{2} = \frac{24}{16}$.

OK

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $\quad 3:x = 24:16$, $x = 2$

DeAndre

I know I need 8 times 3 to get 24, so I have to multiply by 8 on the bottom, too. 16 times 8 is 128, so *x* is 128.

Pitfall

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $\quad 3:x = 24:16$, $x = 128$

Things to Remember

* .....

.....

* .....

.....

STOP

Solve for *x* in Proportions

NAME:

Our Turn

Find the value of *x*. Be prepared to discuss your method.

1. $20:14 = 30:x$ $x =$ _____

2. $35:x = 7:25$ $x =$ _____

3. $x:10 = 30:25$ $x =$ _____

NAME:

My Turn

Find the value of x. Be prepared to discuss your method.

1. $14:5 = 42:x$ $x =$ _____

2. $24:x = 16:6$ $x =$ _____

3. $x:7 = 5:14$ $x =$ _____

Solve for *x* in Proportions

NAME:

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. $x:5 = 12:30$

○ 2 ○ 6 ○ 180 ○ 72

2. $3:10 = x:15$

○ 2 ○ 4.5 ○ 8 ○ 15

STUDENT PAGE 5

Solve for *x* in Proportions

NAME:

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. $x:5 = 12:30$

○ 2 ○ 6 ○ 180 ○ 72

2. $3:10 = x:15$

○ 2 ○ 4.5 ○ 8 ○ 15

NAME:

Writing Task Mini Lesson

Explain how you know that *x* equals 5 in the following proportion.
You may make a diagram to help you explain.

$$15:12 = x:4$$

Solve for *x* in Proportions STUDENT PAGE 6

NAME:

Writing Task Mini Lesson

Explain how you know that *x* equals 5 in the following proportion.
You may make a diagram to help you explain.

$$15:12 = x:4$$

Solve for *x* in Proportions

Lesson at a Glance

Prior Learning Needed

- Understand the concepts of ratio and proportion
- Use mental math and cross multiplication to solve proportions

Lesson Preparation

- Study Lesson Foundation
- Review Teaching Guide and Student Pages
- Prepare stapled packet of Student Pages 1–4 for each student
- Copy and cut in half Student Pages 5 and 6
- Post *Discussion Builders* poster

Mathematical goals

✱ Use scale factors to solve proportion problems

✱ Consider the relationship between division and missing factor problems

Mathematical language and reasoning goals

✱ Write proportions horizontally and in fraction form

✱ Explain the steps used in solving for a missing value in a proportion

LESSON ROADMAP			MATERIALS
CORE LESSON: DAY 1	GROUPING	TIME	❍ *Discussion Builders* poster ❍ Projector (optional) ❍ Student Pages 1 and 2 ❍ Teaching Guide ❍ Scratch paper, calculators (suggested)
Opener *Discussion Builders* Purpose Math Words			
Starter Problem			
Discussion Student Thinking			
Things to Remember Reflection			
CORE LESSON: DAY 2			❍ Clipboard Prompts, page 37 ❍ Student Page 2 (completed day 1) ❍ Student Pages 3 and 4 ❍ Teaching Guide ❍ Scratch paper, calculators (suggested)
Review and Practice Review Day 1 Lesson			
Our Turn			
My Turn			
MINI LESSONS: 2–3 DAYS LATER			❍ Student Pages 5 and 6 ❍ Teaching Guide ❍ Scratch paper, calculators (suggested)
Assess and Reinforce Multiple Choice Mini Lesson			
Writing Task Mini Lesson			

Lesson Foundation

LESSON SNAPSHOT

Starter Problem

Find the missing number to make the ratios equal.

$3:x = 24:16$

Student Thinking

Amina

I rewrote the proportion in fraction form. The scale factor is 8, since 3 times 8 equals 24. Since x times 8 has to equal 16, x must equal 2. It checks since $\frac{3}{2} = \frac{24}{16}$.

OK

$\frac{3 \cdot \boxed{8}}{x \cdot \boxed{8}} = \frac{24}{16}$ $\quad 3:x = 24:16$, $x = 2$

DeAndre

I know I need 8 times 3 to get 24, so I have to multiply by 8 on the bottom, too. 16 times 8 is 128, so x is 128.

Pitfall

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $\quad 3:x = 24:16$, $x = 128$

MATHEMATICAL INSIGHTS & TEACHING TIPS

Proportions and Proportional Relationships

A proportion is a statement of equality between two ratios. A proportional relationship has one base ratio and infinitely many equal ratios, including the two in a proportion problem. Just like with equal fractions, the numbers in one ratio can be scaled to find the numbers in other equal ratios. Amina rewrote the proportion ($3:x$ and $24:16$) using fractions ($3/x = 24/16$) and then worked with fractions to find the missing value. Mathematically, multiplying both numbers in a ratio by the scale factor 8 is like multiplying the entire ratio by a form of 1, or 8/8, giving an equivalent ratio.

Pitfall DeAndre recognized the factor of 8 that relates 3 to 24, but mistakenly multiplied 16 by 8 rather than solving "x times 8 equals 16."

Another method for solving proportions is to use cross products to write an equation. This method will be introduced in a later lesson after students understand how to solve simple proportions by thinking about scale factors.

Lesson Foundation (continued)

MATHEMATICAL INSIGHTS & TEACHING TIPS (CONTINUED)

Prompt students to discover that the cross products in a proportion are equal. Ask them to find the product of the first term of one ratio and the second term of the other ratio, and then compare it to the product of the other two ratio terms. If the products (i.e., cross products) are equal, then the ratios form a proportion.

Scaling Up or Scaling Down for Equal Ratios

DeAndre correctly recognized that the scale factor 8 relates the numbers in the first ratio to the numbers in the second ratio, but he used the scale factor inconsistently. Since he multiplied 3 by 8, he needed to find x by thinking of "what number times 8 equals 16?" Instead, he multiplied 8 by 16. His answer didn't make sense because the first ratio was equal to a fraction less than 1, whereas the second was greater than 1. Estimating the size of x before calculating could help prevent such an error.

To solve the proportion, you could divide the numbers in the second ratio by 8 (i.e., $24 \div 8 = 3$ and $16 \div 8 = 2$, so $x = 2$). There is also a consistent relationship between the first and second numbers in each ratio in a proportion. In the ratio 24:16, for example, 24 is 1.5 times 16, so the missing number times 1.5 should equal 3.

Keeping the Relationship Between the Two Ratios Consistent

When a scale factor is not a whole number, a common pitfall is to look for a whole number relationship between any two numbers in the proportion and use it for the scale factor. So, to solve the proportion $3:4 = x:6$ for x, students may mistakenly think the scale factor is 2, since 3 times 2 is 6. Instead, the scale factor is 1.5, since $4 \bullet 1.5 = 6$.

MATHEMATICAL DISCUSSION SUPPORT

Ask students to explain what it means "to solve for x" in an equation. Discuss how solving proportions is similar to working with fractions. For example, they may suggest simplifying 24/16 to the ratio 3/2. The solution is then obvious since $3/x = 3/2$.

Have students make up a word problem to go with the proportion problem. Sample problem: There are 24 dogs and 16 cats at the animal shelter. This is equal to a ratio of 3 dogs for every x cats. Find the missing number x.

English Learner Access

Ask students to read equations involving proportions using language such as "3 is to x as 24 is to 16"; "3 compares to x as 24 compares to 16"; and "3 for every x is equal to 24 for every 16." Note that it doesn't make sense to say 3 out of x is equal to 24 out of 16, but it does make sense to say 3 out of 4 people is equal to x people out of 240 people.

Core Lesson Day 1

Opener

Review *Discussion Builders*

Read the poster. **Suggest** a section to focus on today:
Presenting Alternative Ideas, Expanding on Others' Ideas, or *Posing Additional Questions.*

Purpose

Distribute stapled packets of Student Pages 1–4. **Project** an image of page 1 (optional).

Call on a student to read the purpose.

Math Words

Point to and say the first math word. **Ask** students to repeat it aloud or silently.

Read the sentence containing the word.

Give an example using objects or drawings.

Repeat for the other math words.

Starter Problem

Read the Starter Problem. **Call on** a student to restate it in his/her own words.

STUDENT PAGE 1

Purpose

To use scale factors to solve for *x* in proportions

Math Words

proportion — Two equal ratios such as 8:6 and 4:3 form a proportion. We say 8 is to 6 as 4 is to 3.

$8:6 = 4:3 \qquad \frac{8}{6} = \frac{4}{3}$

equation — You can solve a proportion equation to find an unknown part.

$\frac{5 \text{ cans}}{7 \text{ days}} = \frac{15 \text{ cans}}{x \text{ days}}$

scale down — To scale down 30 by a factor of 5, divide 30 by 5.

Starter Problem

Find the missing number to make the ratios equal.

$3:x = 24:16$

Think about what the Starter Problem means. WAIT
Try to use what you understand to solve the problem on your own.

I'll walk around and write notes about things we need to discuss. Look out for pitfalls! WAIT

Look at your work. It's easy to have a pitfall in this type of problem. You might also have made a pitfall if your answer is 3 or greater.

Don't worry. Next we'll discuss how two imaginary students solved this problem. One has a pitfall! You may keep your solution private, but bring up your ideas in the discussion.

Core Lesson
Day 1
(continued)

Discussion

Student Thinking

STUDENT PAGE 2

I rewrote the proportion in fraction form. The scale factor is 8, since 3 times 8 equals 24. Since x times 8 has to equal 16, x must equal 2. It checks since $\frac{3}{2} = \frac{24}{16}$.

OK

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $3 : x = 24 : 16$
$x = 2$

Ask students to refer to page 2. **Read** the statement marked OK.

Explain that this statement is about the same problem students worked on earlier.

We can learn a lot about the math by studying what this student did.

Read each sentence silently and look at Amina's work. Think about what they mean. WAIT

Now talk with a partner about what each sentence and each part of Amina's work means.

Listen in, ask questions, and observe. **Note** potential contributions for the discussion.

Who can come up to show us how Amina rewrote the ratios in the problem?

How would she figure out the scale factor is 8? Explain.

Talk to your neighbor about what Amina meant when she said that "since x times 8 has to equal 16, x must equal 2." WAIT

Explain why x is equal to 2. How can we check? Would simplifying 24/16 help?

What happens if you start with the ratio 24/16 and scale it down by dividing? What number would you divide by? What would x equal?

Call on students to state things to remember about solving problems like this.

Start a Things to Remember list on the board.

MORE DAY 1

Core Lesson
Day 1
(continued)

Discussion

Student Thinking, continued

STUDENT PAGE 2

DeAndre

I know I need 8 times 3 to get 24, so I have to multiply by 8 on the bottom, too. 16 times 8 is 128, so *x* is 128.

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $3 : x = 24 : 16$ $x = 128$

Pitfall

Read the statement marked Pitfall. **Remind** students that this is a common pitfall.

DeAndre made a pitfall when he multiplied 16 times 8 to find the value of *x*. Talk with your neighbor about why this is incorrect. WAIT

Was DeAndre's answer too high or too low? Should both ratios be greater than 1 or less than 1? Explain.

Write the following on the board. **Ask** students to talk with a neighbor about how to solve for *x* in these proportions and to decide if the scale factor will be a whole number or not. **Remind** them to look out for pitfalls. **Call on** students to explain why their answers make sense.

$5:x = 20:32$ and $8:5 = 12:x$

Things to Remember

Call on students to **add** to the Things to Remember list on the board. **Read** the list.

Help students summarize and record two important Things to Remember.

Things to Remember List (sample)

1. You can write ratios in a fraction format and work with them like equal fractions.
2. To solve a proportion, you can multiply or divide both terms of one ratio by the same scale factor to find the terms of the other ratio.

Reflection

Ask students to reflect on the discussion process using one of the sample prompts.

Reflection Prompts (sample)

- Name a *Discussion Builder* that we used today. How did it help the discussion?
- What *Discussion Builder* could we use next time to make the discussion even better?
- What did someone do or say today that helped you understand the math?

Core Lesson Day 2

Review and Practice

Review

Ask students to review page 2 to jog their memory.

Read the statement marked OK. **Call on** a student to explain how the problem was solved.

Read the statement marked Pitfall. **Call on** a student to explain why it is incorrect.

Call on two or three students to read an item on their Things to Remember list.

STUDENT PAGE 2

I rewrote the proportion in fraction form. The scale factor is 8, since 3 times 8 equals 24. Since x times 8 has to equal 16, x must equal 2. It checks since $\frac{3}{2} = \frac{24}{16}$.

OK

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $3 : x = 24 : 16$ $x = 2$

I know I need 8 times 3 to get 24, so I have to multiply by 8 on the bottom, too. 16 times 8 is 128, so x is 128.

Pitfall

$\frac{3 \cdot 8}{x \cdot 8} = \frac{24}{16}$ $3 : x = 24 : 16$ $x = 128$

Our Turn

Ask students to refer to page 3.

Use the procedure below and the Clipboard Prompts to discuss students' solutions. **Discuss** the problems one at a time.

Read the problem.

Ask students to work with a neighbor to solve it.

Discuss one or two students' solutions.

Answer Key
1. 21
2. 125
3. 12

STUDENT PAGE 3

Our Turn

Find the value of x. Be prepared to discuss your method.

1. $20:14 = 30:x$ $x =$ _____

2. $35:x = 7:25$ $x =$ _____

3. $x:10 = 30:25$ $x =$ _____

My Turn

Ask students to solve the problems on page 4. **Remind** them to watch out for pitfalls!

After allowing time to work, **read** the answers. **Have** students use pens to mark and revise their papers.

Answer Key
1. 15
2. 9
3. 2.5

STUDENT PAGE 4

My Turn

Find the value of x. Be prepared to discuss your method.

1. $14:5 = 42:x$ $x =$ _____

2. $24:x = 16:6$ $x =$ _____

3. $x:7 = 5:14$ $x =$ _____

Mini Lessons
(2–3 Days Later)

Assess and Reinforce

Multiple Choice Mini Lesson

Distribute Student Page 5.

Problem 1

Please read problem 1.
Talk with your neighbor about which choices don't make sense. WAIT
How do you know the correct choice must be less than 12?
Who can explain how to find the correct choice of 2?

STUDENT PAGE 5

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. $x:5 = 12:30$

● 2 ○ 6 ○ 180 ○ 72

2. $3:10 = x:15$

○ 2 ● 4.5 ○ 8 ○ 15

Encourage students to check by seeing if multiplying by a scale factor can change both numbers in the ratio with lesser numbers into the other ratio.

Problem 2

Read the problem and find the correct choice. WAIT
Which response is correct? Explain why.
How could you use reasoning to know that the correct choice would not be a whole number?

Writing Task Mini Lesson

Distribute Student Page 6.

Ask a student to read the task. **Call on** students to respond with their ideas.

Jot the ideas on the board.

Write an explanation together using their ideas. **Read** it aloud.

Ask students to write an explanation on their page.

STUDENT PAGE 6

Writing Task Mini Lesson

Explain how you know that *x* equals 5 in the following proportion. You may make a diagram to help you explain.

$15:12 = x:4$

Sample Explanation: If you divide 12 by 3 you get 4, so the scale factor is 3. Then, you divide 15 by the scale factor 3 to get 5. So, *x* is equal to 5. The first ratio is scaled down by a factor of 3 to give the second ratio. It checks because if you simplify 15/12, you get 5/4.

English Learner Access

Mathematical Discussion Support

Ask students to read the proportion and explain in their own words what they are trying to do. Students may find it helpful to rewrite the proportion in fraction form to better see the relationships between the numbers.

Ask students to use terms such as "scale factor," "proportion," "scaled up," and "scaled down" in their explanation.