A Descriptive Analysis of *Math Pathways and Pitfalls* in a Latina/o Bilingual Classroom[1]

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A Descriptive Analysis of *Math Pathways and Pitfalls* in a Latina/o Bilingual Classroom[2]

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This paper describes findings of a qualitative study of the implementation of "Math Pathways and Pitfalls" (MPP) in an urban Bilingual classroom that is predominantly Latina/o. Analysis of videotaped MPP lessons both confirms earlier findings of the positive effects of MPP with bilingual learners (BL) and provides insights of characteristics of mathematics learning environments that support Latina/o students. The discussion links the positive effects of MPP to principles of effective instruction with bilingual learners, and thus, demonstrates how mathematics teaching can incorporate these principles.

The *Math Pathways and Pitfalls* (*MPP*) instructional materials were designed to foster students' abilities to represent and reason about mathematical concepts, and to develop their abilities to present complete and coherent mathematical explanations and arguments (Barnett-Clarke, Ramirez, & Coggins, 2010; Barnett-Clarke, & Ramirez, 2004). It also was assumed that the design of the materials, with its emphasis on models and supports for extended discourse, would support the mathematical practice and understanding and language development of bilingual learners (BLs)[3] even though *MPP* was not specifically targeted toward Latinas/os and other language minority students. Nevertheless earlier observations of classrooms with bilingual

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[3] We have chosen to use the term Bilingual Learners; we use this interchangeably with Latina/o and English Language Learner. For us Bilingual Learner keeps front and center Latinas/os lived experiences with two languages. While not all Latinas/os speak Spanish, a significant number come from homes and communities where Spanish is spoken, which influences students' knowledge and resources.

learners suggested that the changes in instruction that resulted from using *MPP* had benefited these students significantly (Barnett-Clarke, Personal communication, April, 2007).

The purpose of this study of *MPP* implementation is to further explore this assumption by understanding the characteristics of *MPP* that create an environment that can support and foster the learning of Latinas/os. Specifically, we were interested in identifying the characteristics of an *MPP* lesson that correlated with current research on instructional practices known to support both linguistic and conceptual development of bilingual learners (e.g., Dalton, 1998;Valdes et al, 2005).

It has long been held that simply "good teaching" is not enough to reverse the poor achievement of Latinas/os in mathematics (Lockwood & Secada, 1999). Instead learning environments need to be modified to meet the unique learning needs of this population of students. While this assumption seems to hold, there is still a question if mathematics curriculum materials can be developed in such a way to better support Latinas/os, linguistically and mathematically, while simultaneously advancing all students (Pitvorec, Willey, & Khisty, in progress). *MPP* may be an example of the foregoing, and an examination of *MPP*'s characteristics will help us understand more about the design of mathematics curriculum to meet diverse students' needs. It should be noted that our interest is not in the academic achievement benefits of *MPP*, even though this is important; rather in this discussion we are interested in if and how *MPP* supports a more positive learning structure that has the potential to increase Latina/o students' development in all aspects through mathematics.

We based our exploration of this question by identifying and examining connections between *MPP* as it was implemented with Latina/o and research-based characteristics of effective instruction for bilingual learners. We begin our discussion by highlighting key concepts drawn from this research that we used in considering various aspects of *MPP*. We then discuss the findings of our qualitative analysis of a set of *MPP* lessons that took place in a urban working-class neighborhood school that was predominantly Latina/o. We end with concluding thoughts.

Concepts for Efffective Instruction for Bingual Learners

We based our analysis of *MPP* lessons on the considerable research regarding the effective instruction for bilingual learners (see for example, Garcia, 1993; Dalton, 1998; Janzen, 2008; Valdés et al, 2005). While we grounded our thinking in this knowledge base, we did not create specific *a priori* items that we searched for in the data. Instead the knowledge base served as a general guide to help us understand the nature of *MPP* as it relates to bilingual learners.

In general, features of effective classrooms for linguistically and culturally diverse students focus on creating an interactive, student-centered learning context (Garcia, 1991), recognizing that academic learning stems from processes of social interaction. This means that there should be abundant and diverse opportunities for students' speaking, listening, reading, and writing. Within this framework of characteristics of effective instruction for bilingual learners, procedural skills are deemed tools for acquiring knowledge rather than fundamental targets of learning activities. In addition to these concepts, there are other critical features of effective instruction for bilingual learners. Some of these are the following:

- Instruction should capitalize on the linguistic and cultural resources of students. This suggests an environment that values students' bilingual resources and that encourages students to use Spanish (Garcia, 1993; Collier and Thomas, 2004) as they develop English;

- Instruction should include various participation structures so that students are active learners rather than passive recipients, but that still provide access to models of thinking and language use (Dalton, 1998);
- Instruction should emphasize critical thinking and minimize remediation (Dalton, 1998). Bilingual learners tend to over-receive low-level content instruction for many reasons including socially and politically based assumptions, but also on the presumption that higher level content (i.e., problem solving, analysis, etc) may be deemed too linguistically complex for them;
- Learning environments should emphasize a functional approach to language development, which is more effective (Mohan & Slater, 2005). This means that bilingual learners need many opportunities to use the target language in natural communication especially about content ideas. They should engage in dialogue that requires extended language use. The key constructs here are "natural" and "extended" language use;
- Instruction should reflect a multimodal approach to communication (Morales, Khisty, & Chval, 2003). This suggests the simultaneous use of various modes of communication such as gestures, graphics, texts, writing, concrete models, drawings, calculators, etc. In this way, "learning through listening only" and displaying knowledge primarily through speaking is minimized (Khisty, 2002)
- Lastly, considerable attention must be given by the teacher to linguistic characteristics (e.g., syntax) of content (in this case, mathematics) that may pose problems and confusions in meanings to students learning in a second language; and that the second language must be modeled by the teacher in such a way that students can acquire the new meanings (Khisty,

1995; Khisty & Chval, 2002). Linguistic characteristics suggest more than vocabulary or terminology.

Overview of Math Pathways & Pitfalls

The lessons that we analyzed followed the design of *MPP*. In this section, we briefly describe *MPP* lessons to give context to our analysis and discussion. In *MPP*, each lesson includes the following components: 1) an introduction of key words and symbols; 2) discussion about two excerpts of student dialogue, one that contains a correct example of student thinking and another that contains a pitfall or misunderstanding in thinking; 3) teacher-guided and individual practice; and 4) follow-up mini-lessons that reinforce each concept, one requiring responses to multiple choice questions and the other requiring a written explanation of a mathematical idea.

Typically two mathematical arguments are presented on an overhead display while students have identical handouts at their seats. On these, a “cartoon” drawing shows two students interacting over a mathematics problem (See appendix for an example). The picture of each cartoon student has a caption that states a conjecture about strategies for solving the problem. After inviting individual students to read each caption publicly in a whole class discussion format, the teacher poses questions (for example, Why would the fictional student think that?) and instructs students to discuss these in their small groups to determine whether the captions represent a “pathway” (a correct way of thinking about the problem or solution strategy) or a “pitfall” (a typically erroneous way of thinking about the problem or solution strategy). Most importantly, students are asked to justify and explain their reasoning of why one cartoon group is thinking either “pathway” or “pitfall”. While students work in small groups, the teacher walks around observing and monitoring the groups’ interactions. At a later point, individual students report their group’s findings in the whole class discussion, and the teacher prompts other students to restate and explain their peers' reasoning. The teacher will refer students to *MPP*'s Discussion Builders whenever necessary. These are phrases that students have become familiar with and that scaffold their skills at formulating and stating their arguments, claims, and responses to others. We discuss the Discussion Builders further in a later section. As students explain their views on the pathways or pitfalls of the various mathematical arguments, the teacher invites students to restate their peers' arguments and pose alternatives. During whole class discussions, students regularly represent their strategies on the overhead or use a poster-size display related to the problem under discussion. Several daily lessons make up a "unit" around a particular mathematics theme.

The Data and Data Analysis

We examined a total of twelve lessons enacted in two 4^{th} grade classrooms in a large, urban public school. Both classrooms were considered mainstream with the bilingual learners having transitioned from the school's Bilingual Program. One class consisted of 25 students, 5 of who were African American and the rest Latina/o. Their teacher, a U. S. born bilingual Latina, had less than 2 years experience teaching elementary school. Instruction and all student talk occurred in English. The other class also had 25 students, all of whom were Latinas/os of varying levels of Spanish and English proficiencies, and included a few new immigrant students who barely spoke or understood English. Their teacher was also Latina but she was born and educated in Spanish-speaking country and had 9 years of classroom teaching experience. All instruction and whole class discussions in this class occurred in English, although we observed numerous instances of Spanish and hybrid Spanish/English use among students and between students and teacher.

After viewing all 12 lessons (2 different classrooms, 6 lessons each), we chose to concentrate our analysis of the 6 lessons from the all-Latina/o classroom in order to closer examine how the processes embedded in *MPP* might benefit bilingual learners whose home language is Spanish (L1) and who may not be highly proficient in English (L2). We are interested in the relation of L1 discourse in L2 learning and how these processes mediate mathematics learning (i.e., development through *MPP*), and how bilingual learners negotiate mathematical tasks, activities, and meaning-making using *MPP* materials which are written in these students' second language (i.e., English).

Specifically, we considered the following questions: 1) What are the qualities of interactions, dialogue, and speech acts, which either encourage or limit mathematical meaning-making; and, 2) what are the qualities of the students' interactions as they produce written artifacts (calculations, drawings, diagrams, written explanations, etc.)?

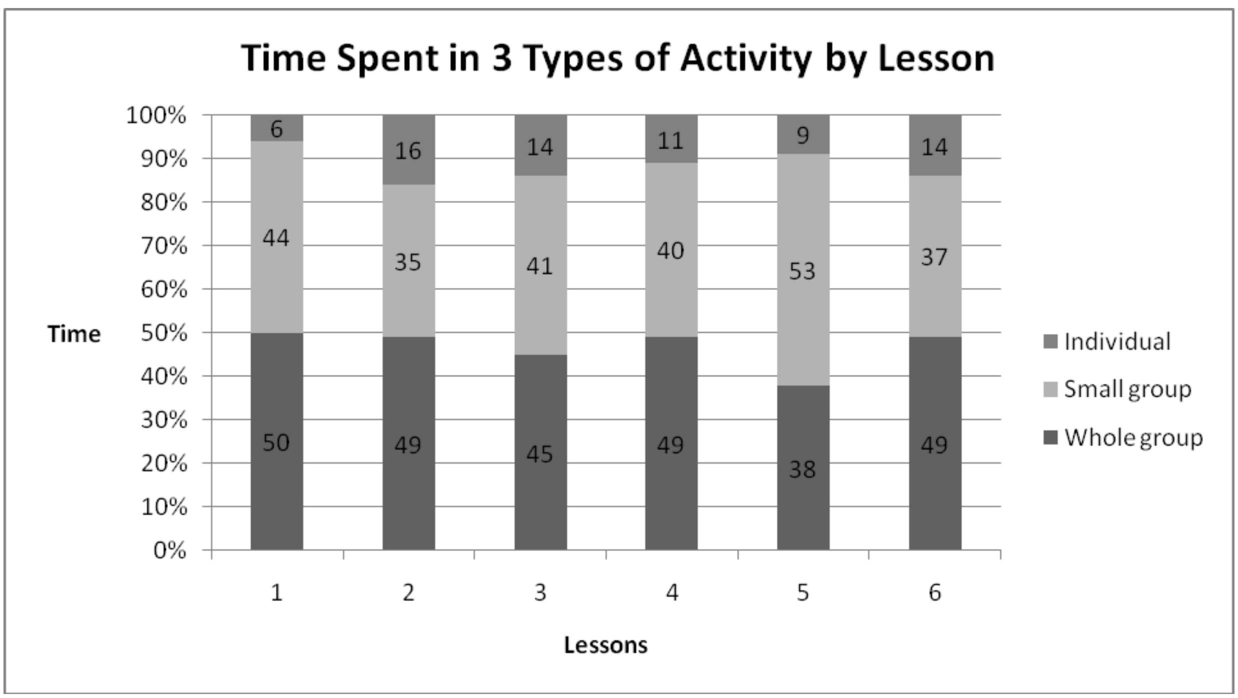
Each lesson was videotaped by two cameras: one stationary and one roving. The roving camera allowed us to see and hear what students were doing during the various segments of small group work while the stationary camera provided a wider perspective that is important for studying whole-class interactions. The videos from these two cameras across six lessons are our primary source of data. Artifacts such as student work were collected but were not always available; when possible and relevant, artifacts were used for triangulation.

Our ethnographic analysis began with an open-ended viewing of all videos for the purpose of refining our constructs and identifying striking elements in the data that we wanted to study more closely (Merriam, 1998). Based on this initial viewing of videos and our questions, we developed a more specific set of constructs, and with these, we looked for patterns which emerged from the data related to students' mathematically productive activity found in the *MPP* lessons. These constructs and patterns which we discuss below relate to 1) the activity structure characteristics of *MPP* lessons (individual, small group, and whole class); 2) the nature of student agency; 3) the presence of real discourse among students; and 4) the use of Spanish . In the following sections we discuss each of these constructs with regard to our findings.

Activity Structures

In our judgment, the use of a variety of activity structures in *MPP* classrooms is the most striking factor that makes *MPP* different from traditional mathematics classrooms. Specifically, the goals, roles of participants, rules of engagement, and mediating artifacts (textbooks, worksheets, English/Spanish use, diagrams, tools, etc.) represent crucial differences between the ways direct instruction techniques or individual problem-solving activities operate and how *MPP* affords opportunities for students to engage in multiple forms of discursive interactions involving mathematical content with peers and their teacher.

Our analysis of the activity structures used during *MPP* revealed that overall, individual activity (mostly quizzes with some individual problem-solving) comprised less than 17% of all five lessons. The remaining classroom time was almost equally divided between whole class and small group activities. While the length of each segment of whole class and small group activity varied for each lesson (sometimes longer sustained sessions of each, sometimes quick transitions between each for shorter periods) there is almost equal distribution of these two formats across the six lessons. For example, in Lesson 5 Day 2, the time distribution for whole class (WC) and small group (SG) activity proceeds as follows: 11 minutes WC, 3 minutes SG, 8 minutes WC, 9 minutes SG, 7 minutes WC, 11 minutes SG, and so on. While this constant shifting may seem chaotic or distracting, it actually seemed to have many positive consequences such as facilitating immediate feedback to students. This shifting of activity structures also provided significant space for students to engage with each other in Joint Productive Activity (JPA) (Dalton, 1998), sharing questions and ideas before discussing them in whole class activity. In typical classrooms, one day or just one portion of a day may be used for any of the common instructional structures. The time between the structures may be too long for the benefits of various forms of dialogue to become effective. The frequent switching between small group and whole class discussions, on the other hand, ensured that students who are otherwise silent during public discussion had the opportunity to practice their reasoning through their second language in less risky situations before sharing their thoughts in front of the teacher and the rest of the class. Also, each item of written text in the *MPP* materials was covered in both formats so that all of the math problems, questions, and concepts were discussed and resolved before moving to subsequent components of each lesson. Shown below in the graph is the percentage of time devoted to whole class, small group, and individual activity for each of the six lessons.



Small group activity is referred to as "partner sharing" in *MPP*. We call these activities "small group" because of the way that it plays out in this particular classroom. Even though the curriculum calls for one-on-one talk between students as they work through math problems, the physical grouping of students' desks often allowed "partners" to interact with other students over the same activity. As students interact with their partners, they often engaged with other students in their cluster.

By switching between whole class and small group activity structures, students have many opportunities to consider their peers' arguments and practice their own explanations in small group activity before exposing their thinking to the more intimidating whole class format. Ideas presented in whole class activity surface again in small groups and vice versa.

Furthermore we often think of scaffolding as some act that, say, the teacher performs especially through her/his own talk or gesture. However, the interplay and movement between whole class and small group activities represents scaffolding just as well:

> "…scaffolding as both structure and process, weaving together several levels of pedagogical support, from macro-level planning of curricula over time to micro-level moment-to-moment scaffolding and the contingent variation of support responsive to interactions as they unfold,
> (Walqui, 2006, p.1). "

Additionally, instead of

> "…simplifying the tasks or the language, teaching subject matter content to English learners requires amplifying and enriching the linguistic and extralinguistic context, so that students do not get just one opportunity to come to terms with the concepts involved, but in fact may construct their understanding on the basis of multiple clues and perspectives encountered in a variety of class activities (Walqui, 2006, p. 169). "

In other words, scaffolding does not proceed in planned, whole-class direct instruction activities or tasks. Instead, this type of guidance emerges between planned and unpredictable events when new or unexpected situations arise. For example, when a student gives an explanation of their mathematical reasoning that would otherwise not be revealed by their paper and pencil work, the teacher or peer must make an effort to understand that student's thinking in order to formulate appropriate guidance. Thus,

> "…scaffolding occurs when planned pedagogical action stops. New and higher skills emerge when new and previously unplanned actions are undertaken. This gives the teacher and learner two separate but interrelated tasks:
>
> - First, structures must be set up to facilitate guided action, since new departures must occur in a safe and familiar context.
> - Second, the teacher and learners must carefully watch for opportunities to depart, expand, elaborate, and improvise, and during those opportunities a handover/takeover must be effected, so that the new emerges from the known, but on the initiative of the learner. (p. 162, van Lier, 2004). "

The structured activity we observed in this *MPP* classroom fulfills all of these conditions. Students enjoyed a safe and familiar context for interaction in their small groups, allowing them space to experiment and improvise with their mathematical arguments before trying their hand at public, whole class discussion. Likewise, the structure and procedures in *MPP* whole class format operated to extend students' thinking as the teacher deferred the role of expert to student contributions, acting as a facilitator to discussion and problem-solving by using phrases such as "can you explain ___'s thinking?" and "what is the pitfall in this argument?" rather than "what is wrong with this student's answer?" If students need to explain their reasoning with the aid of numbers, text, and diagrams, those contributions were valued and encouraged. As these events proceeded, the teacher exploited various "unpredictable" opportunities to scaffold and revoice student contributions, allowing students to publicly rehearse the mathematical language, textual, and diagrammatic conventions necessary for competence in academic discourse. These formats allowed the teacher considerable opportunity to exploit natural language ability in service of academic development. In addition to an expert-novice relationship, scaffolding occurs between equal peers, between a peer to a lower-level peer, and in a self-access, self-regulated situation (van Lier, 2004). In both whole class and small group formats, we found a variety of these expert-novice relationships in action.

Agency

Another important feature of MPP to emerge is in regard to how the presentation and analysis of "pathways" and "pitfalls" fostered students' agency. The use of fictional student voices in *MPP* problems and the task of explaining why a fictional student would think the way she/he does encouraged students to take on the role of "expert" and to offer her/his thinking as a way to help the fictional student. Overall this serves to validate student contributions to the mathematical discourse. This seemly small characteristic of MPP (i.e., that of having students analyze fictional students' thinking) shifts the focus from a single voice of authority (i.e., the author) to an actual vehicle for drawing on students' knowledge and resources.

We view learning as situated in discourse communities and that gaining access to such communities is limited by opportunities to participate in and rehearse these ways of thinking, acting, and speaking. Likewise, the development of student agency is dependent on opportunities for participation. It follows that some participation structures provide better access to or control of tools, resources, and identities necessary for full participation. *MPP* participation structures increase students' opportunities to participate in discourse communities. The excerpt in the next section is also relevant to our discussion of student agency. As this group of students discusses the problem, **8 - .4**, they interrupt each other, finish each others' thoughts, and in general carry on a discussion that is not characteristic of whole class formats where opportunities for interjections and rapid movement between speakers are more constrained. The students are wrestling with the teacher's suggestion to regroup the values but have become distracted by the fact that the whole number and decimal share a common divisor of 2. Student 5 (S5) brings their attention to the decimal point and is in disagreement with the group's strategy of regrouping by twos. Student 5 is usually silent during discussions and is the least English proficient student in this group, a phenomenon common among language learners immersed in mainstream classrooms. The correct strategy involves decomposing the two values in tenths and Student 5 is correct to bring the group's attention to the decimal point ("point the 8") although he has not yet learned how to express well this idea in English. Yet in this activity, he is asserting his mathematical reasoning and *risks* disapproval of his group members' strategy. This shift in his participation and persistence in the face of tension during peer-to-peer communication signals his exercise of personal agency in engaging with mathematical activity.

The following excerpt is another example of how students take on the role of "expert" to help two fictional students. The student refers to the "pitfall" in the text of the lesson handout (Appendix A).

> …and she didn't multiply, no she didn't add and he put the tens *no pero* (no but) ... *tienes que recorder para que no haces* pitfalls *como ella* [you have to remember so you don't do pitfalls like her] and she put 1 and 5 together *si pero* [yes but] things to remember so you don't do pitfalls *como ella* [like her].

In the video, this student seems very animated, shifting in his seat and gesturing with his arms with pencil in hand as he addresses his group members. He refers to the fictional student in the text of the *MPP* handout (Appendix A) in personal terms ("she" and "he"). Although this text serves to anticipate a common mistake students make in this type of problem, we believe that such representations are not necessarily the most important feature of this type of text. The representation of a student/novice explaining a wrong strategy serves to personalize mathematical discourse for students. The text also models ways of presenting and explaining mathematical ideas, and thus, facilitates students' acquisition of mathematical norms. Instead of a disembodied textual voice pronouncing only correct strategies for problem solving, students are exposed to the real nature of mathematical discourse through a personalized representation of a wrong argument elevated to the status of instructional text. This represents a main goal of *MPP* curricula which emphasizes the value of exposing one's thinking and strategies for the sake of discussion and learning.

Real Discourse

During the small group episodes, especially, we observed students engaging in mathematically rich interactions: contesting each other's reasoning, assisting their peers to say something and in reading text, pointing to key portions of diagrams, creating their own representations of the problem, using paper/pencil calculations, asking questions, and extending their reasoning by posing alternate examples. Even during the whole class discussion that was facilitated by the teacher and where a student was presenting her/his argument with the overhead projector, student utterances were rarely single words or phrases. Instead, we noticed complex speech interactions. This is a striking pattern since, generally, instruction with Latinas/os is characterized by students working in silence (Brenner, 1994) and by a teacher's over use of direct instruction that often constrains student talk and that promotes one-word utterances and incomplete sentences (Khisty & Willey, in progress). Both of these characteristics limit students' opportunities to develop advanced language proficiencies since students have little control and decision-making in how to express themselves. On the other hand, the nature of *MPP* with its format of contrasting ways of thinking about a problem solution (i.e., the pathways and pitfalls), deliberately invites students to engage in extended language use and to do so at a linguistically and cognitively higher level. *MPP* encouraged more active roles for students by providing contexts for them to act, think, and speak in ways that traditional mathematics materials limit.

The following excerpt is an example of the type of discourse and complex interaction that we index with our coding strategy. There are five students in this cluster of desks and, as is often the case in real classrooms, one student's partner is absent that day, forcing him to join a pair of partners for this small group activity. This small group of five male students is attempting to solve the problem, **8-.4**, and has just received a hint from the teacher (line 9) that prompts their discussion on how to use regrouping as a strategy to solve the problem.

T represents the teacher; Sts represents all the students in a chorus; S1 through S5 represent individual students.

Lesson 7 Day 1

1. T: Remember our discussion from before? Can we take a whole number from pieces?
2. Sts: (in unison) Uh…no.
3. T: Why not?
4. S4: 'Cause it's not a fraction?
5. T: What is not a fraction?
6. S5: The 4 is a fraction and the 8 is a whole number.
7. T: What are we gonna do? How can we take away a fraction from a whole number?....How can we take away from 8 whole numbers 4 tenths? How could we do that?...
8. S1: Oh I don't get it...
9. T: Well I'm gonna give you a hint...could you regroup over here? Could you regroup? OK, I'm gonna give you a minute to think how could you regroup. I'm gonna give you that hint and think with that in mind… regrouping… how can we can regroup? hmmmm…? (teacher walks away to a different group)
10. S3: You're talking like the whole group.
11. S5: But how could we regroup?
12. S4: ummm… four eighths
13. S2: Like 2, 2, 2, and 2 to get into the whole...
14. S1: So like 2 circle and each circle has 4 people, like two tables [draws circles as he speaks.
15. S3: I think you both are saying like this…
16. S2: I'm saying like this, look, 1, 2,3,4,5,6,7,8 then look 1,2,3,4 (points to paper while counting)
17. S3: I think your saying like this one has four, like this, and if we say we have another table…
18. S3: Then we have four more, so you're saying 8.
19. S2: 8 and then we regroup them...

During the few minutes before the teacher's hint, students S1, S2, and S3 dominate the discussion while the less English proficient students, S4 and S5, are noticeably silent. As this exchange progresses, we see that S4 and S5 begin to take a more active role in the discussion. Leading up to this point, we see contestation and tension as the students wrestle with how to implement the teacher's hint to regroup. Their strategy for approaching this hint involves drawing circles on their papers to represent the decomposition of the whole number, 8. They have not yet attacked the problem of decomposing **.4**, which might have led them to a correct strategy for solving the problem

Wrong arguments, or ways of thinking about a problem, were part of the public classroom discourse and the emphasis was on figuring out how and why these arguments from the fictional characters are incorrect. Moreover, correct arguments were also held to the same standard and are publicly interrogated just as much. This approach nicely inculcates students to the idea that to question something does not mean that it is wrong.

We also observed that in whole class discussions, students consistently presented diagrams, calculations, and other public displays of information (a routine encouraged by *MPP*). This use of student-generated artifacts in whole class discussion is different from what is often found in non-*MPP* classrooms where students may simply orally present their findings. It also represents other crucial but frequently neglected dimensions of academic discourse development among bilingual learners. The use of diagrams and other displays of information provide bilingual learners multiple ways of conveying their ideas without having to rely solely on speaking. *MPP* curriculum explicitly encourages students to use these multimodal strategies: "Draw a picture" and "talk about your thinking" represent strategies that are emphasized in every lesson, both verbally and in the text.

Furthermore, the materials engaged students in recognizing that pathways to meaning-making equally involve serious consideration of "pitfalls" or wrong approaches. A typical instructional focus based entirely on identifying the "correct method" or "correct way of thinking about a problem gives students an incomplete view of authentic mathematical practices. In addition, students had to engage in higher order thinking such as analysis and evaluation to decide why something is a pitfall or a pathway, and this thinking came about through the extended talk between and among students. Again, the emphasis on dialogue and discussion puts students in charge of their learning and speaking experience and creates structured spaces for students to practice how to use language in combination with mathematical practices.

While each lesson in the data set offered examples of students engaging in more extended and complex language use, we also noted that students did not speak this way completely on their own. In our analysis, we observed many instances where students used the *MPP* Discussion Builders in a fairly natural way (i.e., they had begun to internalize the habit of speaking using the Discussion Builders), although some students still seemed awkward using them and some phrases were used more than others. The *MPP* Discussion Builders provided linguistic support for participating in discussions and for maintaining the social norms that facilitate this discussion. They are a set of rules, procedures, and sentence stems of phrases and questions students are encouraged to use during whole class and small group discussions. The Discussion Builders are posted in a large display, visible from all sections of the classroom and students have their own copies at their seats. The following are some of these Discussion Builders:

<u>Presenting Alternative Ideas</u>
I have an idea.
I wonder what would happen if…
I have a counterexample.
I have a conjecture. What if we tried…?

<u>Expanding on Others' Ideas</u>
I have a question about ___'s idea.
I'm confused about…
I'd like to add to ___'s idea.
I agree/disagree with ___'s idea because…
<u>Posing Additional Questions</u>
Would that be true if…?
Is there another way?
Can we think of a counter-example?
How could we prove it?

As can be noted from the above list, the Discussion Builders are clearly more than examples of vocabulary words that bilingual learners should know. Instead they are models of thinking at a higher level (for example: I wonder what would happen if…). These prompts serve as essential scaffolds for students negotiating whole class and small group discussions and the meanings being conveyed. They are, in a sense, rules of engagement intended to help students focus on ideas rather than on the person making a statement. The Discussion Builders, therefore, are tools for scaffolding students' socialization into metacognitive activity. This is particularly important since unsuccessful problem solving is characterized by students' poor metacognitive decisions made worse by a lack of critical engagement with each other's thinking; while successful outcomes are more likely if students challenged and discarded unhelpful ideas and actively endorsed useful strategies (Goos, Galbraith, & Renshaw, 2002).

In scaffolding student learning in such ways as with the Discussion Builders, teachers provide key assistance in a process of socialization where everyday meanings are reshaped into mathematical concepts. This *redescription* (Bartolomé, 1998) is an essential component of mathematical socialization and the process of mathematization is the specific manifestation of *redescription* that occurs in mathematics teaching and learning. It is also an exercise of thinking strategies not usually encouraged in traditional formal schooling with Latinas/os or by rote memorization of procedures that dominates their schooling (Khisty &Willey, in progress).

Use of Spanish in Real Mathematics Discourse

The loss of home language is thought to constrain academic performance (Wong-Fillmore, 2000), especially when ways of thinking and acting using the home language become devalued, effectively eliminating important student resources for meaning-making. Students' use of their L1 allows them to provide each other with scaffolded assistance and to regulate their own cognitive activity. Our observations found that students in our focus classroom frequently spoke in Spanish with each other and with the teacher; however, Spanish was most frequently spoken in the context of doing mathematics. For example, in the following excerpt two students are working on explaining the "pitfall" and the "pathway" for the problem: "Find the sum of .30 + .90". The boy and girl in this example sit at a group of desks with three other students but they address only each other during this short excerpt. The male student watches and listens intently with pencil poised over his paper as the girl draws a diagram while talking, essentially modeling a "think aloud" protocol.

T = Teacher, FS = Female Student

1. T: *It will help you out to solve the problem if you talk.*
2. FS: *Mira, de que otro modo lo podemos hacer? Una bola seria diez que cuesta diez.* [Look, in what other way can we do it? One ball would be ten, which costs ten.] (The girl draws 3 circles putting a 10 in each circle to represent .30. She then draws nine circles on her sheet and puts the number 10 in each circle to represent .90. She shows her partner and says:)
3. FS: *Mira, yo creo que puede hacer así. Mira, los dibujos. Esto es lo que podriamos hacer con los dibujos*. [Look, I believe that it could be like this: Look at the drawings. This is what we could do with the drawings.] (She then counts a total of 12 circles on the paper).
3. FS: *Tenemos doce en total. Nada mas ponemos uno y así porque también tiene el cero acá. Lo podemos hacer doce punto cero y nada mas ponemos el punto. Como un entero y un viente que quedaron.* [We have a total of 12. We place one here like this because it also has the zero here. We could make it 12 dot zero and we just add the dot. Like one whole and a twenty are left.] (She points to the number 120 that she wrote on her paper).

4. FS: *Y para que hagamos mas cortito lo podemos hacer así para que sean enteros, total.* [And to make it shorter we can do it this way so it will be wholes, total.] (The male student is listening.)

The teacher appears again at the end of this short interaction and asks, in Spanish, "Why is this solution correct?" The male student responds in English and successfully explains the problem solution. But during his interaction with the female student immediately preceding the teacher's appearance, both are heard using Spanish exclusively, that is, the girl explains her thinking and the boy listens to it. It appears that the boy has negotiated the mathematical content using Spanish before he articulated his understanding in English. In short, students are allowed and encouraged to make mathematical meaning with whatever resources they are most comfortable. At the same time, they are constantly exposed to contexts (including interactions) that help scaffold their development of mathematical language in both English and Spanish. The types of mathematics questions they are asked to consider naturally extend their linguistic repertoires; students need to listen carefully and to develop arguments to support their thinking or positions. Even though this class is designated by the school system as "mainstream" (meaning that instruction is in English exclusively), we see that it is necessary for both the teacher and students to rely on Spanish for the sake of maintaining real mathematical discourse and problem solving.

In another group of students during this same time frame, we observed two bilingual male students arguing about the same problem, but their language use is different.

20. S5: The teacher said we can *podemos como… podemos hacer estos dos* [we can like, we can do these two] because this one is a fraction and this one is a…
21. S4: …is a whole number
22. S5: It's a whole number that isn't a fraction so how can we do it? How can it be four? We have to think about it 'cause you didn't hear the teacher. We can't have...'cause it doesn't have a point the 8

Here we see they speak in English first, but as the mathematical argument between them continues, they switch to Spanish. In addition, Student 4 finishes Student 5's sentence in the last lines, showing that his thinking is aligned with S5. Also, they use hybrid language, interjecting Spanish words into English sentences and vice versa. They also begin some sentences in English and finish in Spanish and vice versa. Such variations in language use for the sake of generating mathematical meaning are common throughout the lessons we observed, and again, demonstrate students' active use of their resources.

Conclusion

Our purpose has been to understand the characteristics of *MPP* that contribute to Latinas/os' learning of mathematics. We were interested in how the curriculum materials promote or hinder instructional elements that create positive learning environments. Our data set was small but the lessons we qualitatively observed suggested some key characteristics. Overall *MPP* reflected some of the key elements of effective instruction for Latina/o bilingual learners we presented in the beginning of our discussion. However *MPP* extended some of these elements in important ways. We noted that *MPP* made spaces for various mathematical activities and dialogic interactions that essentially require students to speak and listen to each other in different ways; these ways take advantage of the fact that students learn differently in one-on-one interactions. Students were continuously directed to respond to each others' ideas, and they had models that could be used to scaffold their practice in public discussion. Too much of textbook and individual problem-solving activity that involves paper and pencil calculation, ignore the unique relationship between reasoning and speech, not to mention second-language development.

We also noted that *MPP* with its emphasis on analyzing ways to think about a problem and on argumentation creates a context where students can actively engage with others and redefine themselves as experts. In other words, the curriculum enables students to have a "voice" and to have what they say valued by others: this is "agency". For bilingual Latinas/os, this is a huge element since so much of what happens in schools defines them in terms of deficits and non-agentive behaviors.

It is worth noting that three variables in the composition of a language register--field, tenor, and mode--are useful in differentiating *MPP* from traditional mathematics curricula. The *field* in a register can be thought of as the topic of discourse. In general, both *MPP* and all other curricula share the same field, namely, mathematics. However, the *tenor* in *MPP* materials (the relationship of speaker to listener), is different from traditional textbooks in which the author is always positioned as "expert". In *MPP*, the student experiments with the role of expert in trying to discern "pathways" from "pitfalls" presented in the fictional students' arguments. The *mode* of language use by students during *MPP* lessons is both active and reflective. Traditional mathematics texts depend largely on the reader's reflective thought with little emphasis on the reader's active reasoning or dialogic interaction with other mathematics learners or the teacher.

Students need to engage in multiple levels of psychological functions – not just recollection (Khisty & Chval, 2002) in order to become socialized into academic mathematical practices. Therefore, schools need to provide more activities and contexts for students to practice various types of mathematical meaning-making activities such as argumentation, writing, problem-solving, and creation of mathematical artifacts such as graphs, charts, and models. This involves recontextualizing academic concepts in a range of "texts" including speech, writing, action, and image. For Latina/o bilinguals, these activities are crucial for their negotiation of the complex interplay between language and content. The negotiation of meanings from one language to another, and from an everyday language register to mathematical discourse to and from both languages is a complex process (Ron, 1999). In *MPP,* we found that students take more active roles in these activities and have more room to maneuver and share responsibilities.

Lastly, the social organization of learning and forms of mediation available inspire new forms of participation and assistance. *MPP* shifts the focus from a consideration of what strategies teachers should use in mathematics and language instruction to the construction of contexts and activities that will engage students in mathematical discourse and the negotiation of multiple systems of meaning making.

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Appendix A

Textual representation of student arguments in *Math Pathways & Pitfalls*

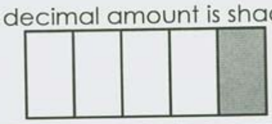
Name ______________________ Date __________ Teacher ________________

Decimals Are Fractions Too

❑ Starter Problem

$\frac{1}{5}$ of this rectangle is shaded.

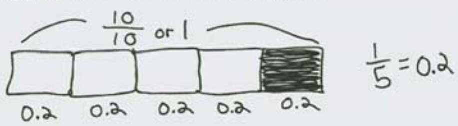
What decimal amount is shaded?



Lee

I needed a decimal, so I called the whole rectangle 10 tenths. There are 10 tenths in all, so each of the 5 parts has 2 tenths. I could have called the rectangle 100 hundredths or 1000 thousandths, too.

OK



Maria

How easy. $\frac{1}{5}$ is the same as point 15. You just put a decimal point in front!

$\frac{1}{5} = .15$

Pitfall

❑ Things to Remember

____

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Student Page 1