Similar Figures

NAME:

Math PATHWAYS & PITFALLS

Purpose

To solve proportion problems involving similar figures

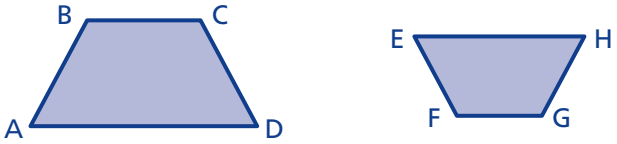
Math Words

similar

The sides of triangle ABC are scaled up by a scale factor of 2 to make a similar triangle EDF.

corresponding sides

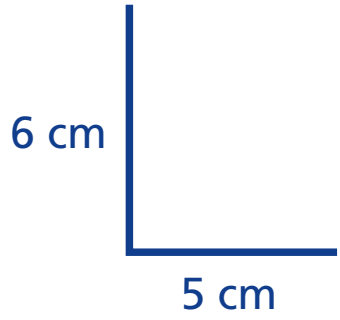
In these similar figures, sides BC and FG are corresponding sides.



Starter Problem

Figure A is similar to Figure B (not shown). The two sides of Figure A are 5 cm and 6 cm long. If the longer side of Figure B is 9 cm long, how long is the other side?

Figure A



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NAME:

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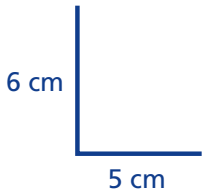
Figure A

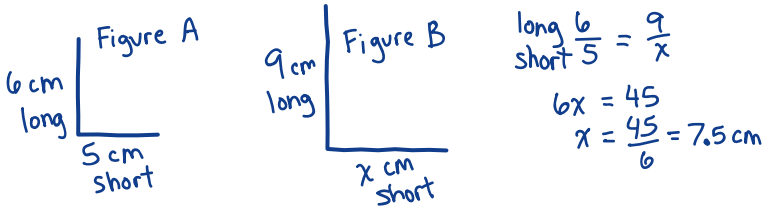


Student Thinking

Maria

Since they're similar, I made a proportion. The first long to short ratio is 6:5. The second long to short ratio is 9:*x*. Then I solved the equation. It makes sense that shorter side is 7.5 cm. If I think of 6:5 and 9:7.5 as fractions, they're both a little more than 1.

OK



Ali

If the long side of Figure B is 3 cm longer than the long side of Figure A, then the short side of Figure B also has to be 3 cm longer than the short side of Figure A. So, 5 + 3 = 8 cm.

Pitfall

Things to Remember

*

*

STOP

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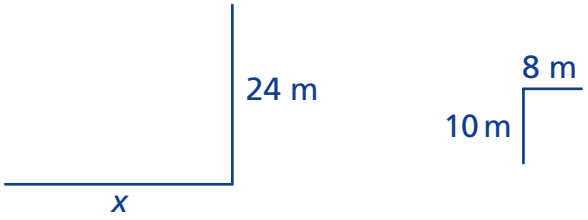
NAME:

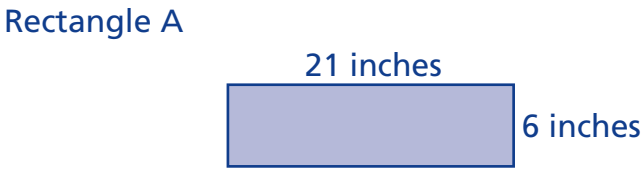
Our Turn

Set up proportions to solve these problems. Draw your own figures if needed. Remember to use labels in your proportions and figures.

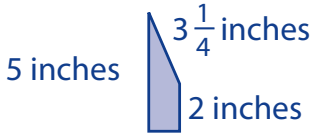
1. The figures below are similar. Find the missing length marked *x*.



2. Rectangle A is 6 inches wide and 21 inches long. Rectangle B is the same shape but bigger. If rectangle B is 20 inches wide, how many inches long is it?



3. A scale drawing of a flowerbed shows two parallel edges that measure 5 inches and 2 inches and a slanted edge of 3 1/4 inches. If the parallel edges of the real flowerbed are 8 yards and 20 yards, how long is the slanted edge of the real flowerbed?



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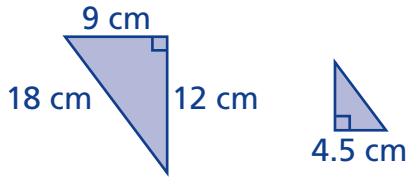
NAME:

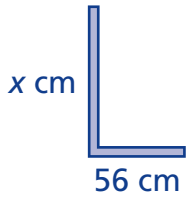
My Turn

Set up proportions to solve these problems. Draw your own figures if needed. Remember to use labels in your proportions and figures.

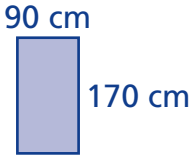
1. The two triangles below are similar. If the shortest side of the smaller triangle is 4.5 cm, how long is its longest side?



2. A large sign was made from an artist's drawing. The measurements of the lines on the letter L on the drawing were 11 cm and 7 cm. The short part of the L on the sign is 56 cm. How long should the long part of the L on the sign be?



3. A door measures 170 cm high by 90 cm wide. If a scale drawing of the door is 4.5 cm wide, how high should the scale drawing be?



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Similar Figures

NAME:

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. The two triangles are similar. How long is the side labeled *x*?

○ 9 units ○ 2 units ○ 4 units ○ 10 units

2. On a scale drawing of a doghouse, the width is 4 inches and the length is 6 inches. If the actual length is 48 inches, what is the actual width?

○ 72 inches ○ 36 inches ○ 46 inches ○ 32 inches

STUDENT PAGE 5 Similar Figures

NAME:

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. The two triangles are similar. How long is the side labeled *x*?

○ 9 units ○ 2 units ○ 4 units ○ 10 units

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○ 72 inches ○ 36 inches ○ 46 inches ○ 32 inches

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NAME:

Writing Task Mini Lesson

Explain how to set up and solve a proportion to show that 40 cm is the correct solution to the following problem.

> A 10 cm by 12.5 cm rectangular picture was enlarged on a copy machine. The longest side of the copy of the picture measures 50 cm. How wide is it?

Similar Figures STUDENT PAGE 6

NAME:

Writing Task Mini Lesson

Explain how to set up and solve a proportion to show that 40 cm is the correct solution to the following problem.

> A 10 cm by 12.5 cm rectangular picture was enlarged on a copy machine. The longest side of the copy of the picture measures 50 cm. How wide is it?

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Similar Figures

Lesson at a Glance

Prior Learning Needed

- Understand the concepts of ratio and proportion
- Use mental math and cross multiplication to solve proportions

Lesson Preparation

- Study Lesson Foundation
- Review Teaching Guide and Student Pages
- Prepare stapled packet of Student Pages 1–4 for each student
- Copy and cut in half Student Pages 5 and 6
- Post *Discussion Builders* poster

Mathematical goals

- ✱ Set up proportions involving similar geometric figures
- ✱ Use cross products and algebra to solve geometric proportion problems

Mathematical language and reasoning goals

- ✱ Discuss similar figures in terms of scaling up and scaling down
- ✱ Use number sense to check proportion problems

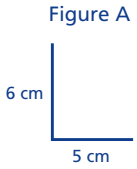
LESSON ROADMAP			MATERIALS
CORE LESSON: DAY 1	GROUPING	TIME	❍ *Discussion Builders* poster ❍ Projector (optional) ❍ Student Pages 1 and 2 ❍ Teaching Guide ❍ Rulers, graph paper, scratch paper (suggested)
Opener			
Discussion Builders Purpose Math Words			
Starter Problem			
Discussion			
Student Thinking			
Things to Remember Reflection			
CORE LESSON: DAY 2			❍ Clipboard Prompts, page 37 ❍ Student Page 2 (completed day 1) ❍ Student Pages 3 and 4 ❍ Teaching Guide ❍ Rulers, graph paper, scratch paper (suggested)
Review and Practice			
Review Day 1 Lesson			
Our Turn			
My Turn			
MINI LESSONS: 2–3 DAYS LATER			❍ Student Pages 5 and 6 ❍ Teaching Guide ❍ Rulers, graph paper, scratch paper (suggested)
Assess and Reinforce			
Multiple Choice Mini Lesson			
Writing Task Mini Lesson			

Lesson Foundation

LESSON SNAPSHOT

Starter Problem

Figure A is similar to Figure B (not shown). The two sides of Figure A are 5 cm and 6 cm long. If the longer side of Figure B is 9 cm long, how long is the other side?

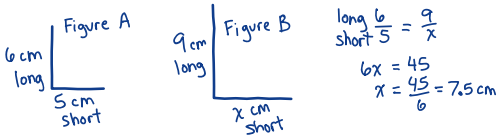


Student Thinking

Maria

Since they're similar, I made a proportion. The first long to short ratio is 6:5. The second long to short ratio is 9:*x*. Then I solved the equation. It makes sense that shorter side is 7.5 cm. If I think of 6:5 and 9:7.5 as fractions they're both a little more than 1.

OK



Ali

If the long side of Figure B is 3 cm longer than the long side of Figure A, then the short side of Figure B also has to be 3 cm longer than the short side of Figure A. So, 5 + 3 = 8 cm.

Pitfall

MATHEMATICAL INSIGHTS & TEACHING TIPS

Similar Figures

Maria understood that when two figures are similar, the length of each side of one figure is multiplied or divided by the same scale factor to give the lengths of the corresponding sides of the other figure. This is why proportions are useful in solving problems involving similar figures. Note that similar figures have the same shape but are not the same size, since one figure is scaled up (or down) proportionally to make the other figure. The corresponding angles in the figures are the same measure.

Pitfall Ali found that one side was 3 cm longer than its corresponding side and mistakenly assumed this relationship would be true for the other pair of corresponding sides.

Maria labeled the terms of her ratio "long to short" and created the ratios 6:5 and 9:*x*. These equal ratios form a proportion that can be solved by the same methods as other proportions. She may have used cross products to solve for *x*. Or, she may have multiplied both sides of her equation by 5*x* in order to clear the denominators, resulting in the equation 6*x* = 45.

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Lesson Foundation (continued)

MATHEMATICAL INSIGHTS & TEACHING TIPS (CONTINUED)

Assign projects in which students start with a simple shape and make many other similar figures by scaling the original figure up and down by various scale factors.

Corresponding Sides in Geometric Figures

When solving geometric proportion problems, only measures for corresponding sides must be compared. When one of a pair of similar figures is rotated to a different orientation, students frequently set up ratios that do not compare corresponding parts. It may be helpful to rotate and redraw one of the figures so that they can be compared in the same orientation. This may be done on transparencies and shown on an overhead projector. A copy machine gives examples of enlarging or shrinking figures by a chosen scale factor.

Subtraction Pitfall with Similar Figures

Ali found the difference in lengths to compare the sides of two proportional figures. However, the ratios of lengths of the sides are based on a multiplicative or division relationship. Students often make this error when the given numbers are close to one another and easy to subtract or when the division isn't easy to calculate.

Ask students to compare the shapes of rectangles after adding or subtracting 2 cm to each side and after multiplying or dividing the length of each side by 2.

MATHEMATICAL DISCUSSION SUPPORT

Ask students questions that prompt them to explain why they wrote their ratios in a particular order. Encourage students to use terms such as "corresponding sides," "similar," and "scaling up" or "scaling down" when talking about the figures. Help students understand that similar figures are proportional because one can be considered a scaled-up version of the other. It may help to show contrasting examples of figures that are not scaled up proportionally.

English Learner Access

When writing proportions on the board or drawing diagrams, ask students to use labels. Make explicit the difference between the mathematical meaning of the word "similar" and the everyday use of the word "similar." Similarity in everyday use does not imply proportionality.

Core Lesson Day 1

Opener

Review *Discussion Builders*

Read the poster. **Suggest** a section to focus on today:
Presenting Alternative Ideas, Expanding on Others' Ideas, or *Posing Additional Questions.*

Purpose

Distribute stapled packets of Student Pages 1–4. **Project** an image of page 1 (optional).

Call on a student to read the purpose.

Math Words

Point to and say the first math word. **Ask** students to repeat it aloud or silently.

Read the sentence containing the word.

Give an example using objects or drawings.

Repeat for the other math words.

Starter Problem

Read the Starter Problem. **Call on** a student to restate it in his/her own words.

Think about what the Starter Problem means. WAIT
Try to use what you understand to solve the problem on your own.

I'll walk around and write notes about things we need to discuss. Look out for pitfalls! WAIT

Look at your work. It's easy to have a pitfall in this type of problem. You might also have made a pitfall if your answer is a whole number.

Don't worry. Next we'll discuss how two imaginary students solved this problem. One has a pitfall! You may keep your solution private, but bring up your ideas in the discussion.

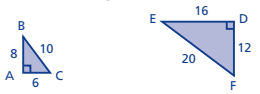
STUDENT PAGE 1

Purpose

To solve proportion problems involving similar figures

Math Words

similar The sides of triangle ABC are scaled up by a scale factor of 2 to make a similar triangle EDF.

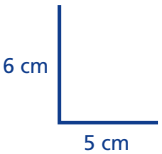


corresponding sides In these similar figures, sides BC and FG are corresponding sides.

Starter Problem

Figure A is similar to Figure B (not shown). The two sides of Figure A are 5 cm and 6 cm long. If the longer side of Figure B is 9 cm long, how long is the other side?

Figure A



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Core Lesson
Day 1
(continued)

Discussion

Student Thinking

STUDENT PAGE 2

Maria

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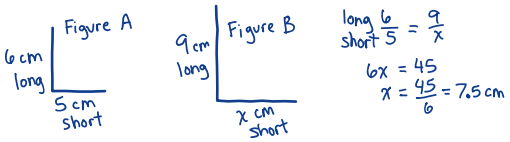
OK



Ask students to refer to page 2. **Read** the statement marked OK.

Explain that this statement is about the same problem students worked on earlier.

We can learn a lot about the math by studying what this student did.

Read each sentence silently and look at the drawing. Think about what they mean. WAIT

Now talk with a partner about what each sentence and each part of the drawing means.

Listen in, ask questions, and observe. **Note** potential contributions for the discussion.

Who can come up and explain how Maria drew Figure B and labeled it? What side in Figure B corresponds to the side in Figure A that is 6 cm long?

Are the lengths in Figure A multiplied or divided by the same scale factor to give the corresponding lengths in Figure B? Is the scale factor greater or less than 2?

Who can come up to explain why Maria's proportion is correct?

Talk to your neighbor about how Maria solved the equation 6/5 = 9/*x*. WAIT

Who can explain the steps Maria used? Explain why her solution makes sense.

Would the proportion 9:6 = *x*:5 also be correct for these figures? What would the labels be? Is the solution the same?

Call on students to state things to remember about solving problems like this.

Start a Things to Remember list on the board.

MORE DAY 1

Core Lesson
Day 1
(continued)

Discussion

Student Thinking, continued

STUDENT PAGE 2

Ali

If the long side of Figure B is 3 cm longer than the long side of Figure A, then the short side of Figure B also has to be 3 cm longer than the short side of Figure A. So, 5 + 3 = 8 cm.

Pitfall

Read the statement marked Pitfall. **Remind** students that this is a common pitfall.

Ali made a pitfall when he thought that the lengths of the corresponding sides of Figure B would be 3 cm longer than in Figure A. Talk with your neighbor about why this is incorrect for similar figures. WAIT

What has to be true about the corresponding sides in similar figures? Explain.

Draw a 3 x 2 rectangle on the board. **Ask** students to talk with a neighbor and show what happens to the rectangle when you add 3 cm to the length of each side versus when you multiply the length of each side by 3. **Remind** students to look out for pitfalls. **Call on** students to show and explain their ideas.

Things to Remember

Call on students to **add** to the Things to Remember list on the board. **Read** the list.

Help students summarize and record two important Things to Remember.

Things to Remember List (sample)

1. We can write proportions for similar figures using ratios of corresponding parts.
2. To make a similar figure, you keep the same angles and multiply or divide the length of each side by the same factor. It doesn't work to add or subtract.

Reflection

Ask students to reflect on the discussion process using one of the sample prompts.

Reflection Prompts (sample)

- Name a *Discussion Builder* that we used today. How did it help the discussion?
- What *Discussion Builder* could we use next time to make the discussion even better?
- What did someone do or say today that helped you understand the math?

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Core Lesson Day 2

Review and Practice

Review

Ask students to review page 2 to jog their memory.

Read the statement marked OK. **Call on** a student to explain how the problem was solved.

Read the statement marked Pitfall. **Call on** a student to explain why it is incorrect.

Call on two or three students to read an item on their Things to Remember list.

STUDENT PAGE 2

Student Thinking


Maria

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OK


Ali

If the long side of Figure B is 3 cm longer than the long side of Figure A, then the short side of Figure B also has to be 3 cm longer than the short side of Figure A. So, 5 + 3 = 8 cm.

Pitfall

Our Turn

Ask students to refer to page 3.

Use the procedure below and the Clipboard Prompts to discuss students' solutions. **Discuss** the problems one at a time.

Read the problem.

Ask students to work with a neighbor to solve it.

Discuss one or two students' solutions.

Answer Key
1. 30 m
2. 70 inches
3. 13 yards

STUDENT PAGE 3

Our Turn

Set up proportions to solve these problems. Draw your own figures if needed. Remember to use labels in your proportions and figures.

1. The figures below are similar. Find the missing length marked *x*.

2. Rectangle A is 6 inches wide and 21 inches long. Rectangle B is the same shape but bigger. If rectangle B is 20 inches wide, how many inches long is it?

3. A scale drawing of a flowerbed shows two parallel edges that measure 5 inches and 2 inches and a slanted edge of 3 1/4 inches. If the parallel edges of the real flowerbed are 8 yards and 20 yards, how long is the slanted edge of the real flowerbed?

My Turn

Ask students to solve the problems on page 4. **Remind** them to watch out for pitfalls!

After allowing time to work, **read** the answers. **Have** students use pens to mark and revise their papers.

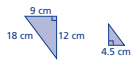
Answer Key
1. 9 cm
2. 88 cm
3. 8.5 cm

STUDENT PAGE 4

My Turn

Set up proportions to solve these problems. Draw your own figures if needed. Remember to use labels in your proportions and figures.

1. The two triangles below are similar. If the shortest side of the smaller triangle is 4.5 cm, how long is its longest side?



2. A large sign was made from an artist's drawing. The measurements of the lines on the letter L on the drawing were 11 cm and 7 cm. The short part of the L on the sign is 56 cm. How long should the long part of the L on the sign be?

3. A door measures 170 cm high by 90 cm wide. If a scale drawing of the door is 4.5 cm wide, how high should the scale drawing be?

Mini Lessons
(2–3 Days Later)

Assess and Reinforce

Multiple Choice Mini Lesson

Distribute Student Page 5.

Problem 1

Please read problem 1.
Talk with your neighbor about which choices don't make sense. WAIT
How could you use reasoning to know that 4 is the correct choice?
Why is it a pitfall to choose 2?

STUDENT PAGE 5

Multiple Choice Mini Lesson

Fill in the circle next to the answer you choose.

1. The two triangles are similar. How long is the side labeled *x*?

○ 9 units ○ 2 units ● 4 units ○ 10 units

2. On a scale drawing of a doghouse, the width is 4 inches and the length is 6 inches. If the actual length is 48 inches, what is the actual width?

○ 72 inches ○ 36 inches ○ 46 inches ● 32 inches

Students may notice that the side marked 8 is 4 units shorter than the corresponding side labeled 12 and mistakenly assume that the side marked x will be 4 units shorter than 6. Remind students that a key concept of similar figures is that corresponding sides are related by multiplication or division, rather than addition or subtraction.

Problem 2

Read the problem and find the correct choice. WAIT
Which response is correct? Explain why.
How could you make a proportion to solve this problem? Explain.

Writing Task Mini Lesson

Distribute Student Page 6.

Ask a student to read the task. **Call on** students to respond with their ideas.

Jot the ideas on the board.

Write an explanation together using their ideas. **Read** it aloud.

Ask students to write an explanation on their page.

STUDENT PAGE 6

Writing Task Mini Lesson

Explain how to set up and solve a proportion to show that 40 cm is the correct solution to the following problem.

A 10 cm by 12.5 cm rectangular picture was enlarged on a copy machine. The longest side of the copy of the picture measures 50 cm. How wide is it?

Sample Explanation: This problem involves similar figures, so the lengths of the corresponding sides make equal ratios: $12.5/10 = 50/x$. The cross products are equal, so 12.5 times x is 500. To solve for x, divide both sides of the equation by 12.5 to get $x = 40$ cm. Also, the scale factor is 4 because 12.5 times 4 is 50. So the missing side is 4 times 10 cm.

English Learner Access

Mathematical Discussion Support

It will help students understand the problem if they diagram and label the similar figures and their dimensions. In addition, they can build mathematical language fluency by verbalizing the steps in the equation using phrases such as "similar figures," "corresponding sides," "solve for *x*," "find the cross products," and "divide each side of the equation by."

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