The Impact of *Math Pathways & Pitfalls* on Students' Mathematics Achievement and Mathematical Language Development: A Study Conducted in Schools with High Concentrations of Latino/a Students and English Learners

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Executive Summary

The purpose of this four-year study was to evaluate the efficacy of *Math Pathways & Pitfalls*, a supplementary K–8 curriculum for students, with professional development for teachers. This discussion-based curriculum has a dual focus on building mathematical concepts and developing mathematical language, with an overall goal of effective and equitable learning.

Using a cluster-randomized experimental design, the study rigorously examined the experimental effects of teachers in grades 4 and 5 using these instructional materials and procedures in place of 15 hours of regular mathematics lessons during each of two academic years. Nearly 70% of the participating students were Latino/a, 55% were classified as English learners, and 75% were eligible for free or reduced-price meals. This study examined the impact of *Math Pathways & Pitfalls* on mathematics achievement and mathematical language development, with special interest in the effects for Latino/a students and English learners. We also assessed the fidelity of implementation and investigated the transfer of *Math Pathways &Pitfalls* classroom practices to regular math lessons (Transfer Study).

The Impact Study was designed to answer two primary research questions:

1. *Program Efficacy.* Do students who use *Math Pathways & Pitfalls* lessons in place of 15 hours of their regular mathematics curriculum exhibit greater increases in *mathematics achievement*, *mathematical language development*, and *English language proficiency* than their counterparts exposed only to regular lessons?
2. *Cumulative Impacts.* Do students exposed to *Math Pathways & Pitfalls* lessons during two academic years (in place of 30 hours of their regular mathematics curriculum) exhibit greater annual improvements in *mathematics achievement*, *mathematical language development*, and *English language proficiency* than their counterparts (a) exposed only to regular lessons, and (b) exposed to *Math Pathways & Pitfalls* lessons during only one academic year (15 hours)?

A volunteer sample of teachers was randomly assigned to experimental and control groups within schools. Study participants included 126 grade 4 and 5 teachers, and over 3,300 of their consenting students, in Arizona, California, and Illinois. Study outcome measures included standardized test scores for mathematics and English language proficiency, as well as project-administered mathematics and mathematical language assessments. Three supplemental data sets were also used: teacher surveys, classroom observation reports, and teacher interview reports.

Overview of Results

Impact study. Hierarchical linear models were used to estimate the effects of different doses of *Math Pathways & Pitfalls* for the full student sample and separately for Latinos, English learners, and English-proficient students. Results showed that just one year (15 hours) of *Math Pathways & Pitfalls* included significant treatment effects, with grade 5 impact stronger than grade 4, and effects were substantially stronger when students had *Math Pathways & Pitfalls* over two years (30 hours). Exposure to the program over two years raised project-administered as well as standardized mathematics test scores for the full sample of students and for the sub-samples of Latino/a students, English learners, and English-proficient students.

One year of *Math Pathways & Pitfalls* raised scores on project-administered mathematics tests for the full sample of grade 5 students as well as for grade 5 Latino/a students and English learners. One year of the program also raised grade 5 students' standardized mathematics test scores, in particular the *Number Sense & Operations* subscale. This significant increase in standardized test scores was found for the full sample of students (effect size .18) and for English-proficient students (effect size .29). In grade 4, impact of one year of *Math Pathways & Pitfalls* on student mathematics achievement was limited to marginally significant increases in English learners' project-administered mathematics test scores (effect size .17), with no other impact on mathematics scores. One year of *Math Pathways & Pitfalls* did not raise Latinos or English learners' standardized mathematics test scores in grades 4 or 5.

The most powerful effects occurred for grade 5 students who also experienced *Math Pathways & Pitfalls* as fourth graders, for the full sample of students as well as for Latino/a students and English learners. Two years of *Math Pathways & Pitfalls,* when compared to one year of exposure, raised both Total Mathematics and *Number Sense & Operations* standardized mathematics test scores, as well as scores on project assessments of mathematics, with effect sizes on standardized tests as high as .29 for Latino/a students and .40 for English learners.

With respect to impact on mathematical language development, one year of *Math Pathways & Pitfalls* had a modest effect, raising grade 4 students' scores on mathematical language, for the full sample of grade 4 students and grade 4 English learners, and had marginally significant effects for grade 4 Latino/a students and the grade 5 full sample. The strongest impact again was evident for grade 5 students exposed to the program for two years. In an analysis comparing no years of exposure to two years of the program, mathematical language scores increased substantially (effect sizes .53-.76) for the full sample of students as well as for the Latino/a, English learner, and English-proficient student sub-groups. Regardless of length of exposure, *Math Pathways & Pitfalls* was not associated with increases in Latino/a students' or English learners' scores on standardized English language proficiency tests.

Implementation fidelity study. Teacher survey data on implementation fidelity indicated that in Year 1, over 90% of the treatment teachers did teach the seven core lessons, with only a few teachers missing at most one of the core lessons or any of the mini lessons. If these self-reported frequencies are accurate, they suggest that a most basic criterion of implementation fidelity was satisfied in Year 1—that the intended number and sequence of lessons be used in treatment classrooms. In Year 2, however, although the treatment group using *Math Pathways & Pitfalls* for the second time reported maintaining their Year 1 level of use and continued implementation of the major components within the lessons, the delayed-treatment control group reported more variability in numbers of lessons used, and also in at least one lesson component. Thus, outcomes for the delayed-treatment control group using *Math Pathways & Pitfalls* for the first time in Year 2 may underestimate the potential impact of the program.

Transfer study. Findings from the exploratory Transfer Study indicated that teachers spontaneously used classroom practices emphasized by *Math Pathways & Pitfalls* in their regular mathematics lessons (non-*Math Pathways & Pitfalls* lessons). A classroom observation rubric documented increases related to mathematics, language and discourse, and equity practices. In surveys, 23.2% of the teachers said that *Math Pathways & Pitfalls* "greatly affected" their teaching of regular math lessons, and 76.8% said that the program affected their teaching of regular math lessons math "slightly" or "somewhat."

Conclusions and Discussion

Results from this efficacy study indicate that exposure to Math Pathways & Pitfalls over two years raised project-administered, as well as standardized mathematics test scores for the full sample of students and for the sub-samples of Latino/a students, English learners, and English-proficient students. When comparing two years of *Math Pathways & Pitfalls* experiences to one year, effect sizes were as high as .40 for English learners and .29 for Latinos on standardized mathematics tests. These findings suggest that *Math Pathways & Pitfalls* adds considerable value to the regular mathematics curriculum in promoting mathematics achievement. The primary conclusions of this study, stated below, are discussed in relationship to the goals and design of the *Math Pathways & Pitfalls* curriculum.

Math Pathways & Pitfalls increases mathematics achievement. Central to each *Math Pathways & Pitfalls* lesson is the unique approach of inviting students to analyze correct and flawed ("pitfall") solution processes. This practice has been shown to lead students to use more correct procedures and remember more correct concepts than only analyzing correct examples (Durkin & Rittle-Johnson, under review). This practice may play an important role in increasing mathematics achievement for *Math Pathways & Pitfalls* students.

Also, *Math Pathways & Pitfalls* aims to develop student's metacognition (self-monitoring or awareness of one's own thinking). The metacognitive behavior prompted by *Math Pathways & Pitfalls* may help students work though mathematical misconceptions and avoid or detect errors. We hypothesize that students transfer metacognitive behavior learned through *Math Pathways & Pitfalls* to their regular mathematics lessons. This would help explain the significant impact of *Math Pathways & Pitfalls* on not only the *Number Sense & Operations* portion of standardized tests, but also the significant impact on the Total Mathematics portion of standardized tests.

Math Pathways & Pitfalls has a positive impact on mathematical language development. All *Math Pathways & Pitfalls* lessons incorporate support for developing mathematical language. For example, lessons engage students in expressing and comprehending mathematical ideas symbolically and verbally, as well as orally and in writing. Teaching guides inform teachers about language issues, key mathematical terms are introduced in context to set the stage for discussion, and a *Discussion Builders* poster scaffolds discussions. Significant increases on the project-designed mathematical language assessment indicate that *Math Pathways & Pitfalls* plays a role in developing mathematical language, which may in turn facilitate mathematics learning.

Math Pathways & Pitfalls raises standardized test scores for both English learners and English-proficient students, as well as Latino/a students. Researchers investigating the implementation of *Math Pathways & Pitfalls* lessons (Khisty & Radosavljević, 2010; Razfar & Leavitt, 2010) found several elements of effective instruction for Latino/a students, English learners, and bilingual students. They include, for example, directing students to respond to others' ideas; scaffolding discussion; emphasis on analyzing ways to think about a problem; enabling students to have what they say valued by others; varied participant structures; and introduction of key mathematical terms for use in discussion. Few studies have reported successful interventions for raising the mathematics achievement of English learner or Latino/a student groups, so these results are especially notable.

The results of this research, combined with findings from an earlier efficacy study in grades 2, 4, and 6 (Heller, Curtis, Rabe-Hesketh, & Verboncoeur, 2007), funded by the National Science Foundation, confirm the positive effects of *Math Pathways & Pitfalls*. These studies provide considerable evidence that *Math Pathways & Pitfalls* increases mathematics learning for students with a variety of economic, geographic, ethnic, and language backgrounds. Future research is needed to understand how *Math Pathways & Pitfalls* influences teaching and supports the adoption of new practices. Ultimately this information can be valuable in designing effective curricula and instruction for students, and professional development for teachers.

Acknowledgments

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In addition to the Principal Investigator, there were two other members of the *Math Pathways & Pitfalls* project staff. They had tremendous responsibilities and pressure during the four years of this research. Alma Ramírez was responsible for overseeing the professional development and training instructors; organizing data collection; and communicating with districts and instructors during the research. Also, Alma's expertise as a bilingual teacher and her personal background as an English learner were important in the development of *Math Pathways & Pitfalls* as well as the research agenda for this study. Kate Darling was responsible for the data management; coordinating the research activities with administrators, researchers, and teachers; and managing the publication process. Her attention to detail and total commitment to quality are greatly appreciated.

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The Impact of *Math Pathways & Pitfalls* on Students' Mathematics Achievement and Mathematical Language Development: A Study Conducted in Schools with High Concentrations of Latino/a Students and English Learners

This report provides an overview of the *Math Pathways & Pitfalls* supplementary curriculum and detailed descriptions of three studies. The *Math Pathways & Pitfalls* Impact Study examines the efficacy of *Math Pathways & Pitfalls* and is the primary focus of the report. A qualitative study was carried out to study the fidelity of implementation. An exploratory investigation, the Transfer Study, is included to consider the possibility that *Math Pathways & Pitfalls* may not only change classroom practices in the few *Math Pathways & Pitfalls* lessons implemented during the study, but also in the regular mathematics lessons as well. In addition, the Summary of Results and Discussion section of this report refers to findings from qualitative studies conducted on the behalf of this project that provide further insight into the possible ways *Math Pathways & Pitfalls* influences teaching and learning.

Math Pathways & Pitfalls

The *Math Pathways & Pitfalls* supplementary curriculum for K–8 students, from which lessons were selected for this research, takes the unique approach of not only fostering correct ways to represent and reason about mathematical concepts, but also explicitly calling students' attention to common errors and misconceptions, which we call pitfalls. *Math Pathways & Pitfalls* also provides lesson-specific assistance for helping students learn how to use mathematical vocabulary and symbols; to present complete and coherent explanations orally and in writing; and to participate in mathematical discourse where they present, analyze, expand, and justify mathematical ideas in paired, small group, and whole class settings.

Math Pathways & Pitfalls was developed and field-tested with grants from the National Science Foundation (ESI 9911374) and Stuart Foundation. The lessons have broad appeal, especially in the existing climate of accountability, since they address some of the toughest math concepts and associated learning pitfalls culled from the research literature and from national and international assessments. The mathematical topics for grades K–3 focus on developing whole number concepts and operations, while the topics in grades 4–8 focus on developing rational number concepts and operations. These intervention materials address the need for improving instruction, regardless of the core instructional materials being used.

The *Math Pathways & Pitfalls* lessons are particularly appropriate for students who are English learners because each lesson includes exercises and tools for building relevant academic and discipline-specific language skills (Heller, Curtis, Rabe-Hesketh, & Verboncoeur, 2007). The English versions of the lessons have shown promising results with students regardless of their language background.

Lesson Structure

Each lesson uses a consistent, easy-to-follow format and includes sections that (a) introduce key words and symbols; (b) promote discussion and reflection about two excerpts of fictitious student dialogue, one that contains a correct example of student thinking and another that contains a pitfall in thinking; (c) provide teacher-guided and individual practice; and (d) reinforce each concept through a pair of follow-up mini lessons, one requiring responses to multiple-choice questions and the other a written explanation of a mathematical idea.

Teaching guides for each lesson provide a mathematical background section and a language support section. The teaching guides also provide a short list of prompts to help teachers conduct a substantive discussion of the key mathematical ideas in the lesson.

In addition to the lesson materials and teaching guide, the *Math Pathways & Pitfalls* supplementary curriculum includes video for building student and teacher meta-cognition about learning and a CD with resources for assessment. That is, the curriculum consists of video, print, and electronic materials, which include: (a) eight units for grades K–8, each with 10–11 core lessons and follow-up mini lessons for students; (b) *Discussion Builders* classroom posters that support math discussions; (c) teaching guides for each lesson, including each mini lesson; (d) videos for teacher professional development and videos for students that model how to present and discuss mathematical ideas; and (e) mathematics assessments correlated to each lesson.

Professional Development

Math Pathways & Pitfalls professional development delivered to participants in this Impact Study used a highly specified foundational curriculum. However, we note that it could be customized to fit different time frames and particular district needs. Experienced instructors, trained by the *Math Pathways & Pitfalls* staff, conducted the professional development institutes and after-school meetings with teachers. The mathematical content of the professional development varies according to the grade level of the teachers and was aligned to *Math Pathways & Pitfalls* lessons for those grades. The grade 4 and grade 5 lessons in this study both focused on fractions and decimals, with a greater emphasis on the meaning of fractions and decimals – particularly equivalence – in grade 4, and a greater emphasis on operations with fractions and decimals in grade 5.

Professional development participants discussed the mathematical concepts targeted in select lessons, ways to represent the concepts, and common misconceptions and pitfalls related to the concepts. Participants read and discussed research related to children's mathematics learning and took part in a practicum focusing on implementing the research-based effective and equitable teaching practices that are embedded in *Math Pathways & Pitfalls*.

Math Pathways & Pitfalls Impact Study

Overview

This four-year research study builds on promising findings from previous research on *Math Pathways & Pitfalls* (Heller, Curtis, Rabe-Hesketh, & Verboncoeur, 2007). The previous work, conducted in schools that were ethnically, geographically, and economically diverse, reported significant positive effects for *Math Pathways & Pitfalls* classrooms compared to non-*Math Pathways & Pitfalls* classrooms on project-designed mathematics assessments after only one year (15 hours) of implementation. That study was conducted in grades 2, 4, and 6 and the lesson focus at grade 2 was addition and subtraction; at grade 4, fractions; and at grade 6, percents.

In contrast to the prior study, the current study was conducted in schools with high ratios of Latino/a students and students designated as English learners. Nearly 70% of the participating students were Latino/a, 55% were classified as English learners, and 75% were eligible for free or reduced-price meals. It was implemented over two consecutive academic years for grades 4 and 5, and each grade level focused on two mathematical topics (fractions and decimals), with the lessons in grade 5 building on the lesson content of grade 4. This efficacy study examined the impact of *Math Pathways & Pitfalls* on students' mathematics achievement as measured by standardized tests and project-designed assessments that were criterion-referenced to the content of the lessons for each grade. Also, the impact of *Math Pathways & Pitfalls* on students' mathematical language development (related to the mathematical content of the set of lessons for each grade) was assessed using project-designed assessments. These effects were compared for different doses of *Math Pathways & Pitfalls*, ranging from no exposure, to one year (15 hours over seven lessons), to two years (30 hours) of exposure.

Data were disaggregated to examine whether the effects were different for English learners, Latino/a students, or English-proficient students. Additional qualitative studies were conducted to assess the fidelity of implementation and help explain the results.

Research Questions

The major aim of the study was to test the efficacy of *Math Pathways & Pitfalls* for improving student performance in mathematics and mathematical language development. We investigated the following questions:

1. Program Efficacy

   Do students who use *Math Pathways & Pitfalls* lessons in place of 15 hours of their regular mathematics curriculum exhibit greater increases in *mathematics achievement, mathematical language development,* and *English language proficiency* than their counterparts exposed only to regular lessons?

2. Cumulative Impacts

   Do students exposed to *Math Pathways & Pitfalls* lessons during two academic years (in place of 30 hours of their regular mathematics curriculum) exhibit greater annual improvements in *mathematics achievement*, *mathematical language development*, and *English language proficiency* than their counterparts (a) exposed only to regular lessons, and (b) exposed to *Math Pathways & Pitfalls* lessons during only one academic year (15 hours)?

In testing these questions, the study relied on measures of mathematics and mathematical language learning, as well as English language proficiency. The research included a cluster-randomized experimental design that allowed us to make causal inferences regarding the effectiveness of the *Math Pathways & Pitfalls* curriculum. In addition, survey data on implementation fidelity were used to validate quantitative findings.

Treatment and Counterfactual

Teachers randomly assigned to be in the treatment condition were requested to participate in 30 hours of professional development (22 hours over four days during the summer of 2006 and 8 hours during the 2006/07 school year) to support their implementation of *Math Pathways & Pitfalls*. During the 2007/08 school year they attended 8 hours of after-school meetings to review *Math Pathways & Pitfalls* lessons and do research tasks such as completing surveys. Teachers assigned to the treatment condition were expected to replace 15 hours of their regular mathematics curriculum with 15 hours of *Math Pathways & Pitfalls* lessons during each academic year in 2006/07 and 2007/08.

In contrast, teachers randomly selected to be in the delayed-treatment control condition were asked to implement their regular mathematics curriculum during the 2006/07 academic year and attend 8 hours of after-school meetings for mathematics activities unrelated to *Math Pathways & Pitfalls* and for research tasks such as completing surveys.

During the second year of the study, teachers assigned to the delayed-treatment control group were requested to participate in 30 hours of professional development to support their implementation of *Math Pathways & Pitfalls* (22 hours during the summer of 2007 and 8 hours during the 2007/08 school year). During the 2007/08 school year, both the treatment and delayed-treatment control groups were asked to replace approximately 15 hours of their regular mathematics curriculum with the *Math Pathways & Pitfalls* lessons.

Research Design

A *true, group-randomized experimental design* was used to control for most threats to internal validity. Teachers were randomly assigned to one of two groups – a *treatment group* that would implement *Math Pathways & Pitfalls* lessons during both Year 1 and Year 2 of the study (2006/07 and 2007/08 academic years), and a *delayed-treatment control group* that would implement the lessons in Year 2 only (see Table 1).

Table 1.
Math Pathways & Pitfalls Experimental Design for Grade 4 and Grade 5 Teachers Randomly Assigned to Treatment or Delayed-Treatment Control Group

	Baseline 2005/06		Year 1—2006/07				Year 2—2007/08		
Teacher Group	Spring	Summer	Fall	Winter	Spring	Summer	Fall	Winter	Spring
				Grade 4					
Treatment	O_S	PD	O_T O_S	MPP	O_S O_T		O_S	MPP	O_S O_T
Delayed-treatment control	O_S	TxU	O_T O_S	TxU	O_S O_T	PD	O_S	MPP	O_S O_T

Teacher Group	Baseline 2005/06 Spring	Summer	Year 1—2006/07 Fall	Winter	Spring	Summer	Year 2—2007/08 Fall	Winter	Spring
				Grade 5					
Treatment	O_S	PD	O_T O_S	MPP	O_S O_T		O_S	MPP	O_S O_T
Delayed-treatment control	O_S	TxU	O_T O_S	TxU	O_S O_T	PD	O_S	MPP	O_S O_T

O_T = Teacher observations or measurement points

O_S = Student observations or measurement points

PD = Professional Development in *Math Pathways & Pitfalls*

MPP = Classroom implementation of *Math Pathways & Pitfalls*

TxU = Treatment as Usual Condition

This study was conducted in schools and classrooms with large concentrations of English learners whose home language is Spanish. To be included in the study, schools must include grade 4 and/or grade 5, with 50% or greater Latino/a students and high concentrations of English learners. Eight districts in three states (Arizona, California, and Illinois) met these criteria and agreed to serve as research sites for this project. Each site agreed to provide access to schools, teachers, and students who met our criteria for participation. They also agreed to provide the project with access to district-administered student achievement test scores and English language proficiency test scores. Each of the eight districts also had a district-level or university-affiliated person assigned to coordinate the logistics of the research study.

The target population for this two-year experimental study was grade 4 and grade 5 students in the classrooms of up to 150 teachers, with roughly equal numbers of classrooms in each school for each grade level. These grade levels were chosen in part because consecutive grades were needed to investigate the cumulative impact of using the lessons for two years. At each collaborating school, we opened recruitment for all teachers who met the specified criteria for the targeted student population.

In each district, project representatives met with potential teacher participants in small groups prior to their voluntary enrollment to discuss the benefits and requirements for their participation. Teachers were paid a stipend based on an hourly rate that was customary for their district. All teachers were requested to participate in a professional development institute targeted specifically to the mathematical concepts and pedagogical strategies of the *Math Pathways & Pitfalls* lessons they were to implement at their grade level. Teachers assigned to the *Math Pathways & Pitfalls* treatment group received their *Math Pathways & Pitfalls* professional development (about 22 hours) in the summer of 2006, prior to the intervention during the 2006/07 academic year along with approximately 8 hours of academic-year follow-up professional development. Delayed-treatment control group teachers received the same professional development the following summer and school year.

Overall, 152 teachers met eligibility criteria and were randomly assigned to experimental groups. Of those 152 teachers, 13 moved or transferred prior to the implementation year and 13 withdrew voluntarily, leaving 126 teachers to participate in core research activities, along with over 3,300 of their consenting students.

Description of the Intervention

The intervention took place during the academic years of 2006/07 and 2007/08 with students in grades 4 and 5. The mathematical focus of the lessons selected for both grades was on fraction and decimal concepts and operations. Consecutive grade levels were chosen so that the lessons could build on the mathematical content presented the previous year, allowing us to test the cumulative growth in content over two consecutive years. By targeting these particular grade levels, we were able to use some of the instruments and professional development materials that were developed for a prior study.

Teachers implementing the intervention taught their regular mathematics curriculum but replaced 15 hours with *Math Pathways & Pitfalls* lessons. The 15 hours was experienced by students in each grade through seven specifically-sequenced core *Math Pathways & Pitfalls* lessons selected for their grade level, each with two follow-up mini lessons on the same topic. The lessons were presented following roughly the same schedule, with a set (core lesson and two mini lessons) presented about every four weeks, beginning in September and ending in May. Each core lesson was presented during two 45-minute math periods and each mini lesson took approximately 15 to 20 minutes of instructional time. Students in the treatment and control conditions had about the same number of instructional hours over the school year. The total amount of instructional time replaced during the school year was 15 hours for those in the treatment condition. This short amount of time was sufficient to have a measurable impact in previous studies.

Only English versions of the *Math Pathways & Pitfalls* lessons were used since one of the key research questions involved impact of the lessons on English learners' mathematical language development in English. However, teachers were not restricted from using language other than English or allowing English learners to speak in their native language during the lessons, especially during paired discussions. The only constraints were that the print and video materials used were in English. The rest of the mathematics curriculum was taught as it would be normally.

Professional Development

The summer before implementing *Math Pathways & Pitfalls*, each teacher was requested to participate in a 22-hour professional development institute specifically designed to support the implementation of *Math Pathways & Pitfalls* at their grade level. In addition, teachers were asked to attend 8 hours of after-school meetings during each year of implementation. The purpose of these meetings was to provide follow-up support for implementation and to handle research-related tasks such as completing questionnaires. During Year 1, teachers in the delayed-treatment control group attended 8 hours of meetings after school where they engaged in mathematical activities unrelated to *Math Pathways & Pitfalls* and completed research-related tasks. The total number of hours of professional development to support the *Math Pathways & Pitfalls* intervention was approximately 30 hours.

Each professional development summer institute drew from relevant research and from examples of classroom practice. The institutes also provided specific mathematical background related to the instruction of the mathematics topics appropriate for the particular grade level. In addition, teachers learned about language issues specific to the mathematical content at their grade and participated in a practicum to learn how to use *Math Pathways & Pitfalls* with their students. As part of the institute, teachers viewed and analyzed a video showing an example of a *Math Pathways & Pitfalls* lesson in action. The institute materials were carefully constructed to be replicated with fidelity and were successfully field-tested by professional development facilitators who had been trained by *Math Pathways & Pitfalls* staff. Dr. William Yslas Vélez, mathematics professor at the University of Arizona, was present during two days of a summer institute to evaluate the mathematical integrity of the training. The instructor's guide for the institute was specifically designed to support the implementation of *Math Pathways & Pitfalls*.

Random Assignment Procedures and Intervention Schedule

The teachers' group assignment and treatment schedule is described in Table 2. Teachers in each grade, within each school, were randomly assigned to either the immediate treatment group or delayed-treatment control group. Teachers in the treatment groups at each grade implemented *Math Pathways & Pitfalls* in both Year 1 and Year 2. Teachers in the delayed-treatment control group delivered no treatment in Year 1, but implemented *Math Pathways & Pitfalls* in Year 2. All grade 4 and grade 5 students in participating teachers' classrooms, regardless of the students' language background, were invited to participate in the lessons. A letter was sent home with each student describing the study and asking for signed consent to participate from both a parent/guardian and the student. The same letter was used across sites and was available in English, Spanish, and 16 additional languages. Only students who returned signed letters indicating consent to participate were included in the study. Students were not randomly assigned to classrooms but rather received whichever treatment their teachers were assigned. Analytical procedures recommended by Raudenbush (1997) and Murray (1998) were followed to account for the probability that students might be clustered within classrooms.

Table 2.
Classroom Mathematics Instruction Delivered in Years 1 and 2 by Grade 4 and Grade 5 Teachers in Each Group

Teacher group assignment	Year 1	Year 2
Treatment group	*Math Pathways &Pitfalls*	*Math Pathways & Pitfalls*
Delayed-treatment control group	No treatment	*Math Pathways &Pitfalls*

Only the students of teachers assigned to the treatment group received *Math Pathways & Pitfalls* lessons in Year 1. During Year 2, all students received the *Math Pathways & Pitfalls* lessons, regardless of their teacher's assignment during Year 1. The study was not conducted in grade 3 or grade 6, so no students were exposed to the *Math Pathways & Pitfalls* lessons during these grades. Thus, as shown in Table 3, students received zero, one, or two years of *Math Pathways & Pitfalls* lessons depending upon the sequence of classrooms they were in.

Table 3.
Number of Years Students Received Math Pathways & Pitfalls in Years 1 and 2

Student's participation in Year 1	Student's participation in Year 2	No. years student received *Math Pathways & Pitfalls*
None (Grade 3 not in study)	Grade 4—treatment	1 year
Grade 4—treatment	Grade 5—treatment	2 years
Grade 4—no treatment	Grade 5—treatment	1 year
Grade 5—treatment	None (Grade 6 not in study)	1 year
Grade 5—no treatment	None (Grade 6 not in study)	0 years

Several steps were taken to ensure that the delayed-treatment control group was not contaminated. First, face-to-face meetings were held at the beginning of Year 1 with both the delayed-treatment control group teachers and the teachers using *Math Pathways & Pitfalls* lessons. In these meetings, *Math Pathways & Pitfalls* representatives discussed the nature of the study and talked with teachers about their professional obligations regarding giving the lessons a fair test. Contamination issues were not considered to be a high risk in this study, since the lesson materials are very specific and would be difficult to share through discussion with a colleague; however, teachers received explicit instructions to prevent sharing. To emphasize the importance of this request, both treatment and delayed-treatment control teachers were asked to sign an affidavit not to talk about or share the materials. Also, teachers in the delayed-treatment control condition were asked survey questions to detect possible contamination.

Participation as Randomly Assigned

Table 4 presents information about teacher participation in the *Math Pathways & Pitfalls* summer institutes by treatment status and by year. Recall that treatment teachers were assigned to attend the *Math Pathways & Pitfalls* summer institute in 2006, while teachers in the delayed-treatment control group were assigned to attend in 2007. Table 4 shows that 89% of the treatment teachers and 75% of the delayed-treatment control teachers participated in the institute to which they were randomly assigned, including one teacher in each group who attended two days of the four-day institute. Note that not all teachers in each group abided by their random assignment status. In Year 1 of the study, four delayed-treatment control teachers crossed over to participate as if they were treatment teachers, attending the 2006 summer institute and teaching *Math Pathways & Pitfalls* lessons. There were seven treatment group teachers who crossed over to participate as if they were delayed-treatment control teachers: four of the seven attended the 2007 institute and taught *Math Pathways & Pitfalls* lessons only in Year 2 of the study. To preserve random assignment, all impact analyses were based on teachers' original random assignment status, regardless of whether they crossed experimental conditions.

Table 4.
Number of Teachers Participating in Math Pathways & Pitfalls Summer Institutes as Randomly Assigned

Summer institute participation (n=126)	Summer 2006				Summer 2007			
	Treatment		Delayed-treatment control		Treatment		Delayed-treatment control	
	Count	%	Count	%	Count	%	Count	%
Summer institute participation								
Full (4 days)	62	87.3	4	7.3	4	5.6	40	72.7
Partial (2 days)	1	1.4	0	0.0	0	0.0	1	1.8
Total participation	63	88.7	4	7.3	4	5.6	41	74.5
Summer institute non-participation								
No participation (0 days)	8	11.3	51	92.7	67	94.4	14	25.5
Total	71	100.0	55	100.0	71	100.0	55	100.0

Methods

Instruments

The evaluation team collected data using multiple methods among the same participants across three years of the study. The primary student outcome measures were student performance in mathematics, mathematical language, and English language proficiency. We also collected data from teachers to monitor the fidelity of implementation and to monitor conditions in control-group classes. Table 5 below summarizes the data collection timeline.

Table 5.
Data Collection Measures and Schedule

Measure	Baseline 2005/06	Year 1 2006/07	Year 2 2007/08
Student outcome measures			
Mathematics achievement			
Project-administered mathematics assessment		Fall/Spring	Fall/Spring
Standardized achievement tests	Spring	Spring	Spring
Mathematical language assessment		Fall/Spring	Fall/Spring
Standardized English language proficiency tests	Spring	Spring	Spring
Fidelity/teacher practice measures			
Teacher surveys		Fall/Spring	Fall/Spring
Classroom observations		Fall/Spring	
Teacher interviews		Spring	

Students' Mathematical Achievement

Students' mathematical achievement was assessed with two measures: (1) district-administered standardized achievement tests and (2) project-administered mathematics assessments.

Standardized mathematics achievement test. Students' mathematics achievement data from district-administered standardized tests for all students were collected for the years before and during the intervention. The participating school districts administered different standardized tests in mathematics. Arizona schools administered Arizona's Instrument to Measure Standards test (AIMS), California schools administered California Standards Test (CST), and Illinois schools administered the Illinois Standards Achievement Test (ISAT).

The standardized tests have been well validated and have good psychometric properties. Although the tests in Arizona, California, and Illinois measure the same general constructs, they are different in terms of content emphasis, item sampling, item difficulty, and the populations from which they are normed (Feuer, Holland, Green, Gertenthal, & Hemphill, 1999). These tests are also not vertically aligned across grades. To convert the scores to an identical metric so that test score data from all of the sites can be analyzed together, all the test score data were normalized within each grade by subtracting the district mean from each student's score and dividing by the district standard deviation (at baseline). This is analogous to techniques used in meta-analysis to pool the results of studies using alternative measures of similar constructs. Normalized in this way, the test score data represent the relative ranking of students within a district rather than the absolute level of mathematics performance, and the impact estimates (see below) reflect the within-district, standardized effect estimate. Because we randomize classes within schools, our impact estimates will not be adversely affected. *Math Pathways & Pitfalls* impacts on (a) Total Mathematics performance and (b) *Number Sense & Operations* subscores were examined.

Mathematics assessments. The two 35-item *mathematics assessments*, one for grade 4 and one for grade 5, were developed by the principal investigator. Items were developed, piloted, and tested for reliability under this grant and National Science Foundation grant ESI 9911374. Identical pre- and post-tests of students' mathematics content knowledge were used for each grade (see Appendices A and B). The assessments demonstrate adequate reliability (α = 0.81 and .88 for grades 4 and 5, respectively). The mathematics assessment items assess fraction and decimal concepts that are known to cause difficulty for students as identified from the research literature and prominent assessments such as the NAEP and TIMMS. The items are in multiple-choice format with at least one of the choices containing a common misconception or error that students have with regard to the concept being assessed.

Students' Mathematical Language Development

Data were collected to assess students' skills in mediating across mathematical words, representations, and symbol systems using instruments developed, piloted, and tested for reliability under this grant and a supplement to National Science Foundation grant ESI 9911374. The *mathematical language development assessments* (see Appendices C and D) were aligned to the mathematics lessons for grade 4 and grade 5. Each consisted of 30–32 multiple-choice items (α = 0.81 and .84 for grades 4 and 5, respectively). In addition, students completed an open-ended question, which provided interesting information for formative uses; however, the focus of this final report are the results from the multiple-choice items. The mathematical language exam was administered concurrently with the mathematics assessment.

Students' English Language Proficiency

Similar to the state standardized mathematics data, English learners' standardized English language proficiency data were collected for the years before and during the intervention for each cohort from district-administered standardized tests for English learners. Because the schools were in different states, they administered different English language proficiency standardized tests. Schools in Arizona administered AZELLA (Arizona English Language Learner Assessment), schools in California administered the CELDT (California English Language Development Test), and schools in Illinois administered the ACCESS English language development test (Assessing Comprehension and Communication in English State-to-State). The same procedures were used to convert the scores to an identical metric (to have a mean of zero and standard deviation of one) so that test score data from all of the sites could be analyzed together. Impacts were assessed for (a) overall English language proficiency, (b) listening, and (c) speaking.

Teacher Background and Beliefs

Treatment and delayed-treatment control group teachers were surveyed at the beginning of Year 1 (pre-assessment) (see Appendices E and F) and at the end of Years 1 and 2 (post-assessment) (see Appendices G–I). Pre-assessment surveys collected information about teachers' background and experience, such as: previous mathematics teaching experience, type of teaching certification, undergraduate and graduate education in mathematics, mathematics education, and methods of teaching English learners. These surveys also collected information about teachers' professional development experiences pertaining to mathematics content and mathematics teaching, strategies for teaching English learners, analyzing student work, instructional materials, and local and state standards, as well as engaging students in talking about subject matter content. Both the pre- and post-assessment teacher surveys also included items relating to teacher beliefs about mathematics teaching and learning along with items about their self-perceptions about their mathematics teaching practices and classroom activities.

The post-assessment surveys completed by both groups of teachers contained questions about the value of the *Math Pathways & Pitfalls* project for them as teachers, their ways of using *Math Pathways & Pitfalls* materials in the classroom, and for information on additional professional development that the teachers participated in during the intervention year.

Teachers in the Year 1 treatment condition also completed an additional survey on their implementation of *Math Pathways & Pitfalls* lessons and the impact of *Math Pathways & Pitfalls* on their classrooms and students (see Appendix G). This additional survey captured their experience teaching *Math Pathways & Pitfalls* lessons in Year 1. In Year 2, all teachers were given the same post-assessment survey, since all had implemented *Math Pathways & Pitfalls* lessons that year (Appendix I).

Sample Characteristics

Overall, 152 teachers met eligibility criteria and were randomly assigned to treatment conditions. After random assignment, 26 of the 152 teachers either moved or transferred to other schools/grades (13 teachers) or withdrew from the study (13 teachers) – leaving 126 teachers who participated in the core research activities in some manner. This group of 126 teachers included 69 grade 4 teachers and 57 grade 5 teachers. These teachers are represented in Table 6, which shows the number of teachers and students who participated in the study by grade and experimental condition. Note that greater numbers of teachers, particularly grade 5 teachers, were in the treatment group than in the delayed-treatment control group. This imbalance is due to three factors: (1) chance imbalances within each school arising from assigning an “odd” number of teachers to an “even” number of conditions,[1] (2) assignment imbalances due to having incorrect teacher grade information at the time of random assignment, and (3) greater attrition after random assignment among delayed-treatment control group teachers than treatment group teachers. Post-random assignment attrition is responsible for about one-third of the imbalance among grade 5 teachers; chance factors seem be responsible for the remainder. Analytic sample sizes reported in other parts of this report may differ slightly from those listed in Table 6 because of missing data.

Table 6.
Numbers of Participating Teachers and Students by Grade and Treatment Status

Sample	Treatment	Delayed-treatment control
Teachers		
Grade 4	36	33
Grade 5	35	22
Students		
Year 1		
Grade 4	713	792
Grade 5	669	373
Year 2		
Grade 5	488	351

The imbalance of grade 5 teachers in both Year 1 and Year 2 is the primary reason why the grade 5 student samples were smaller in the delayed treatment group than in the treatment group. Fewer grade 5 teachers were available to serve Year 2 students who had been promoted from grade 4 treatment classrooms.

Table 7 presents demographic characteristics and pre-intervention test scores of students participating in the study. The table shows that approximately 55% of participating students were classified as English learners, where a student was defined as an English learner if the district had ever categorized the student as an English learner during the study period. Nearly 70% of students were Latino/a, and 75% were eligible for free or reduced-price meals. Consistent with the aims of recruitment, the study sample is comprised of high proportions of students who traditionally exhibit below-average academic performance. It is important to note that the treatment and delayed-treatment control groups were almost identical on all baseline characteristics, as would be expected with random assignment of teachers to groups.

[1] For example, if three grade 5 teachers participated in a school, two would have been randomly assigned to one group and one would have been randomly assigned to the other group.

Table 7.
Student Baseline Characteristics - Analytic Sample

Characteristic	Treatment[A]	Delayed-treatment control[A]	Difference	*p*	Difference/SD
English learner	0.56	0.54	0.02	.62	0.05
Free/reduced-price meals	0.76	0.76	-0.00	.88	-0.01
Race/ethnicity					
African American	0.08	0.08	-0.01	.57	-0.03
Asian	0.04	0.04	0.00	.60	0.03
Latino	0.71	0.67	0.04	.26	0.08
White	0.07	0.10	-0.03	.35	-0.16
Female	0.50	0.53	-0.03	.18	-0.05
Standardized test scores					
Mathematics 05 (std)	-0.03[B]	0.04[B]	-0.06	.40	-0.09
Overall English proficiency 05 (std)	0.05[B]	-0.04[B]	0.10	.53	0.13
Mathematics Assessment Pretest					
Grade 4	0.24[C]	0.26[C]	-0.03	.09	-0.22
Grade 5	0.32[C]	0.30[C]	0.02	.34	0.12
Mathematical Language Assessment Pretest					
Grade 4	0.35[C]	0.37[C]	-0.03	.18	-0.18
Grade 5	0.41[C]	0.37[C]	0.04	.06	0.15

Notes. *p*-values are based on multilevel regression models in which treatment group status is included as a covariate.
[A] Values are proportions unless otherwise noted.
[B] Average standardized test scores (mean = 0. standard deviation = 1).
[C] Average proportion of test items answered correctly.

Data Analysis Methods

Table 8 describes the comparisons that were made and the outcome measures used to address each research question. Refer to Tables 2 and 3 for a description of the teacher and student groups compared. To examine the extent to which students exposed to one year of *Math Pathways & Pitfalls* lessons exhibited greater increases in *mathematics achievement*, *mathematical language development*, and *English language proficiency* than their delayed-treatment control-group counterparts exposed only to regular lessons, we estimated achievement differences between students in treatment and delayed-treatment control classes after the first year of *Math Pathways & Pitfalls* implementation. Separate models were estimated for grade 4 and grade 5 students. To ascertain whether students exposed to *Math Pathways & Pitfalls* for two academic years obtained more benefit from 30 hours of exposure than their delayed-treatment control group counterparts exposed only to regular lessons, we estimated achievement differences between grade 5 students in Year 2 who were exposed to *Math Pathways & Pitfalls* as grade 4 students in the prior academic year, and grade 5 delayed-treatment control students in Year 1 who never experienced the lessons (note that this comparison did not involve randomized groups). We also estimated achievement differences between grade 5 students who were exposed to *Math Pathways & Pitfalls* during two academic years, and their grade 5 counterparts who were exposed to 15 hours of *Math Pathways & Pitfalls* during only one academic year (Year 2).

Table 8.
Contrasts Addressing Research Questions

Targeted outcomes	Groups compared	Outcome measures
Question 1: Impact of one year vs. no exposure to *Math Pathways & Pitfalls*	Treatment vs. delayed-treatment control classrooms in Year 1 in grade 4 and Year 1 in grade 5	• Standardized mathematics achievement test • Mathematics assessment • Mathematical language assessment • Standardized English language proficiency test
Question 2a: Impact of two years vs. no exposure to *Math Pathways & Pitfalls*	Grade 5 students in Year 2 who received treatment in both grades 4 and 5 vs. grade 5 students in Year 1 delayed-treatment control*	• Standardized mathematics achievement test • Mathematics assessment • Mathematical language assessment • Standardized English language proficiency test
Question 2b: Impact of two years vs. one year of exposure to *Math Pathways & Pitfalls*	Grade 5 students in Year 2 who received treatment in both grades 4 and 5 vs. in grade 5 only	• Standardized mathematics achievement test • Mathematics assessment • Mathematical language assessment • Standardized English language proficiency test

*This comparison is between groups that were not randomly assigned.

Because students were nested within teachers and teachers were nested within schools, the primary hypothesis-testing analyses involved fitting linear mixed effects ANCOVA models (HLM and multilevel models), with additional terms to account for the nesting of subjects within units of aggregation (e.g., see Goldstein, 1987; Raudenbush & Bryk, 2002; Murray, 1998). Random effects include teacher to account for within-site clustering. Potential fixed effects include treatment group, baseline (pretest) measures of outcome variables, school, and other observed covariates. The following model was estimated for each student achievement outcome:

$$Achieve_{ij} = \alpha_0 + \beta_1 Tx_j + \beta_2 \mathrm{Pre}Achieve_{ij} + \sum \beta_I I_{ij} + \sum_{S=2}^{S} \beta_S School + \mu_j + \varepsilon_{ij}$$

In this model, subscripts *i* and *j* denote student and teacher, respectively; *Achieve* represents student achievement; *Tx* is a dichotomous variable indicating teacher assignment to the treatment group; Pre*Achieve* is a baseline measure of the outcome measure, assessed prior to random assignment; *I* represents a set of comparison variables (grade, ethnicity, SES, and gender); *School* represents a set of dichotomous variables for schools (randomization strata); and μ and ε represent random error terms. The intervention effect represented by β_1 is of primary interest, as it represents the treatment/control class difference in annual change in student performance between pretest and posttest. μ captures random effects of teacher, which account for positive intraclass correlations in the data. Note that the model is appropriate for evaluating both the short-term (research question 1) and cumulative (research questions 2a & 2b) impacts of student exposure to the *Math Pathways & Pitfalls* lessons. To preserve random assignment and account for self-selection, impact analyses included all students with valid outcome data, whether or not their *Math Pathways & Pitfalls* teachers actually delivered the materials in their classrooms or agreed to participate in the study (an intent-to-treat analysis).

A primary focus of the research is on the effectiveness of the lessons on the mathematics achievement of English learners and Latinos. Extensions to the model described above have been made to examine how *Math Pathways & Pitfalls* lessons impact student learning among students in these groups. For example, including an interaction term between *Tx* and indicators of English learner status captured differences in effectiveness for English learners from others.

Results of Impact Analyses

One-Year Math Pathways & Pitfalls Impacts

Table 9 shows impact estimates pertaining to research question 1, comparing outcomes for students who had *Math Pathways & Pitfalls* lessons (treatment in Year 1) versus students who had no exposure to the program (delayed-treatment controls in Year 1). Table 10 shows impact estimates separately for students classified as Latinos, English learners or English-proficient.

Table 9.
Intent-to-Treat Impact Estimates—Math Pathways & Pitfalls vs. Usual Curriculum for Full Student Sample (One Year vs. No Exposure)

Measure	Treatment: One year of MPP	Delayed-treatment control: No MPP	Difference	*p*	Effect size (Diff/SD)
		Grade 4_a			
Mathematics Assessment [A]	0.40	0.38	0.02	.28	0.09
Mathematics Standardized Tests[B]					
Total Mathematics	0.06	0.02	0.04	.67	0.04
Number Sense & Operations	0.01	0.07	-0.06	.57	-0.06
Mathematical Language Assessment[A]	0.51	0.47	0.04**	.04	0.18
English Language Proficiency[B]	-0.01	0.13	-0.14*	.08	-0.14
Listening	-0.05	0.16	-0.20***	.01	-0.20
Speaking	0.12	0.11	0.02	.86	0.02

Measure	Treatment: One year of MPP	Delayed-treatment control: No MPP	Difference	*p*	Effect size (Diff/SD)
		Grade 5b			
Mathematics Assessment[A]	0.48	0.44	0.04**	.03	0.19
Mathematics Standardized Tests[B]					
Total Mathematics	0.03	-0.06	0.08	.31	0.08
Number Sense & Operations	0.05	-0.11	0.17**	.04	0.18
Mathematical Language Assessment[A]	0.53	0.50	0.03*	.09	0.15
English Language Proficiency[B]	0.05	0.05	0.00	.97	0.00
Listening	-0.02	0.10	-0.12	.25	-0.13
Speaking	0.07	-0.07	0.15	.35	0.15

Note. Data are regression-adjusted using multilevel regression models that include baseline test scores, demographic characteristics, and randomization strata as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.

[a] Treatment: classroom *n* = 35, student *n* = 641; delayed-treatment control: classroom *n* = 34, student *n* = 628.

[b] Treatment: classroom *n* = 31, student *n* = 563; delayed-treatment control: classroom *n* = 21, student *n* = 328.

*Significantly different from zero at the 0.10 level, two-tailed test. **Significantly different from zero at the 0.05 level, two-tailed test. ***Significantly different from zero at the 0.01 level, two-tailed test.

[A] Average proportion of test items answered correctly.

[B] Average standardized test scores (mean = 0. standard deviation = 1).

Table 10.
Intent-to-Treat Impact Estimates for Latinos and by English Learner Status—Math Pathways & Pitfalls vs. Usual Curriculum (One Year vs. No Exposure)

Measure	Treatment: One year of MPP	Delayed-treatment control: No MPP	Difference	*p*	Effect size (Diff/SD)
		Grade 4a			
Mathematics Assessment[A]					
Latinos	0.39	0.37	0.02	.31	0.11
English Learners	0.39	0.36	0.03*	.10	0.17
English-Proficient Students	0.40	0.40	0.00	.97	0.00
Mathematics Standardized Tests[B]					
Total Mathematics					
Latinos	0.01	-0.01	0.01	.89	0.01
English Learners	0.12	0.04	0.08	.41	0.08
English-Proficient Students	-0.02	-0.01	-0.01	.92	-0.01
Number Sense & Operations					
Latinos	-0.02	0.02	-0.04	.69	-0.05
English Learners	0.04	0.04	0.00	.99	0.00
English-Proficient Students	-0.02	0.11	-0.13	.29	-0.13

Measure	Treatment: One year of MPP	Delayed-treatment control: No MPP	Difference	*p*	Effect size (Diff/SD)
Mathematical Language Assessment[A]					
Latinos	0.50	0.46	0.04*	.06	0.16
English Learners	0.50	0.46	0.05**	.02	0.23
English-Proficient Students	0.51	0.49	0.03	.24	0.12
English Language Proficiency[B]					
Latinos	-0.05	0.09	-0.14*	.08	-0.14
English Learners	-0.01	0.13	-0.14*	.08	-0.14
Listening					
Latinos	-0.10	0.11	-0.21***	.01	-0.20
English Learners	-0.05	0.16	-0.20***	.01	-0.20
Speaking					
Latinos	0.08	0.07	0.01	.92	0.01
English Learners	0.12	0.11	0.01	.86	0.02
Grade 5_b					
Mathematics Assessment[A]					
Latinos	0.48	0.44	0.04**	.02	0.20
English Learners	0.45	0.42	0.04*	.06	0.20
English-Proficient Students	0.50	0.46	0.04**	.03	0.19
Mathematics Standardized Tests[B]					
Total Mathematics					
Latinos	0.02	-0.05	0.08	.31	0.08
English Learners	-0.06	-0.06	0.01	.96	0.01
English-Proficient Students	0.10	-0.04	0.14	.13	0.14
Number Sense & Operations					
Latinos	0.01	-0.11	0.13	.15	0.14
English Learners	-0.11	-0.14	0.03	.77	0.04
English-Proficient Students	0.20	-0.08	0.28***	.01	0.29
Mathematical Language Assessment[A]					
Latinos	0.53	0.50	0.03	.11	0.16
English Learners	0.49	0.46	0.03	.19	0.16
English-Proficient Students	0.57	0.53	0.03	.13	0.16

Measure	Treatment: One year of MPP	Delayed-treatment control: No MPP	Difference	*p*	Effect size (Diff/SD)
English Language Proficiency[B]					
Latinos	0.02	0.04	-0.02	.81	-0.02
English Learners	0.05	0.05	0.00	.97	0.00
Listening					
Latinos	-0.04	0.10	-0.14	.21	-0.14
English Learners	-0.02	0.10	-0.12	.25	-0.13
Speaking					
Latinos	0.04	-0.06	0.10	.53	0.10
English Learners	0.07	-0.07	0.15	.35	0.15

Note. Data are regression-adjusted using multilevel regression models that include baseline test scores, demographic characteristics, and randomization strata as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.

[a] Treatment: classroom $n = 35$, student $n = 388$; delayed-treatment control: classroom $n = 34$, student $n = 628$.

[b] Treatment: classroom $n = 31$, student $n = 298$; delayed-treatment control: classroom $n = 21$, student $n = 252$.

*Groups are significantly different at the 0.10 level, two-tailed test. **Groups are significantly different at the 0.05 level, two-tailed test. ***Significantly different from zero at the 0.01 level, two-tailed test.

[A] Average proportion of test items answered correctly.

[B] Average standardized test scores (mean = 0. standard deviation = 1).

Overall, the results shown in Tables 9 and 10 for one year of exposure to *Math Pathways & Pitfalls* versus no exposure indicate the following:

Grade 4 Mathematics

- Project assessment: *Math Pathways & Pitfalls* is not associated with significant increases in scores on the mathematics assessment for grade 4 students, but there are marginally significant increases for English learners exposed to *Math Pathways & Pitfalls* compared with English learners in the delayed-treatment control group ($p = .10$, $ES = .17$).
- Standardized assessment: *Math Pathways & Pitfalls* is not associated with increases on mathematics standardized test scores for grade 4 students.

Grade 4 Language

- Project assessment: *Math Pathways & Pitfalls* is associated with increases in scores on the mathematical language assessment for the full sample of grade 4 students ($p < .05$, $ES = .18$) and for English learners ($p < .05$, $ES = .23$). Results on the mathematical language assessment were marginally significant for grade 4 Latinos ($p < .10$, $ES = .16$).
- Standardized assessment: *Math Pathways & Pitfalls* is not associated with increases in scores on standardized English language proficiency tests for grade 4 Latinos or English learners.

Grade 5 Mathematics

- Project assessment: *Math Pathways & Pitfalls* is associated with increases in scores on the mathematics assessment for the full sample of grade 5 students ($p < .05$, $ES = .19$), for Latinos ($p < .05$, $ES = .20$), and for English-proficient students ($p < .05$, $ES = .19$). This treatment-control difference is marginally significant for English learners ($p < .10$, $ES = .20$).
- Standardized assessment: *Math Pathways & Pitfalls* is associated with increases in standardized mathematics test scores on the *Number Sense & Operations* subscale for the full sample of grade 5 students ($p < .05$, $ES = .18$) and for English-proficient students ($p = .01$, $ES = .29$) but not for Latinos or English learners.

Grade 5 Language

- Project assessment: There is a marginally significant treatment-control difference on the mathematical language assessment for the full sample of grade 5 students ($p < .10$, $ES = .15$).
- Standardized assessment: *Math Pathways & Pitfalls* is not associated with increases in scores on standardized English language proficiency tests for grade 5 Latinos or English learners.

Cumulative Impacts – Two Years vs. No Exposure to Math Pathways & Pitfalls

Tables 11 and 12 show results pertaining to research question 2a, focusing on the cumulative impacts of two years' student exposure to *Math Pathways & Pitfalls* as compared to no exposure. These results indicate that two years of *Math Pathways & Pitfalls* raise student scores on all measures in the study, with the exception of English language proficiency. The results for two years of exposure to *Math Pathways & Pitfalls* versus no exposure indicate the following:

Grade 5 Mathematics

- Project assessment: Two years of exposure to *Math Pathways & Pitfalls* increases scores on the mathematics assessment, as compared to no exposure, for the full student sample ($p < .01$, $ES = .64$), as well as for Latinos ($p < .01$, $ES = .65$), English learners ($p < .01$, $ES = .72$), and English-proficient students ($p < .01$, $ES = .58$).
- Standardized assessment: Two years of exposure to *Math Pathways & Pitfalls* increases standardized mathematics test scores, as compared to no exposure, for the full student sample both for Total Mathematics scores ($p < .01$, $ES = .29$) and the *Number Sense & Operations* subscale ($p < .01$, $ES = .36$). These differences are also significant for Latinos ($p < .05$, $ES = .25$, and $p = .01$, $ES = .32$ for total and subscale scores, respectively), English learners ($p < .05$, $ES = .34$ and .37 for total and subscale scores, respectively), and English-proficient students ($p = .05$, $ES = .25$ and $p = .01$, $ES = .37$).

Grade 5 Language

- Project assessment: Two years of exposure to *Math Pathways & Pitfalls* increases scores on the mathematical language assessment, as compared to no exposure, for the full student sample ($p < .01$, $ES = .63$), as well as for Latinos ($p < .01$, $ES = .65$), English learners ($p < .01$, $ES = .76$), and English-proficient students ($p < .01$, $ES = .53$).

- Standardized assessment: Two years of exposure to *Math Pathways & Pitfalls,* as compared to no exposure, is not associated with increases in scores on standardized English language proficiency tests for grade 5 Latinos or English learners.

Table 11.
Intent-to-Treat Impact Estimates—Effects of Grade 4 Exposure to Math Pathways & Pitfalls on Grade 5 Student Outcomes for Full Student Sample (Two Years vs. No Exposure)

Grade 5 Measure	Treatment: Two years of MPP[a]	Delayed-treatment control: No MPP[b]	Difference	*p*	Effect size (Diff/SD)
Mathematics Assessment[A]	0.57	0.42	0.15***	.00	0.64
Mathematics Standardized Tests[B]					
Total Mathematics	0.16	-0.13	0.29***	.00	0.29
Number Sense & Operations	0.17	-0.18	0.34***	.00	0.36
Mathematical Language Assessment[A]	0.60	0.47	0.13***	.00	0.63
English Language Proficiency[B]	0.01	-0.04	0.04	.87	0.05
Listening	-0.04	0.02	-0.06	.86	-0.06
Speaking	-0.03	-0.05	0.03	.90	0.03

Note. Data are regression-adjusted using multilevel regression models that include baseline test scores, demographic characteristics, and randomization strata as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.

[a] Classroom $n = 44$, student $n = 296$.

[b] Classroom $n = 20$, student $n = 328$.

*Groups are significantly different at the 0.10 level, two-tailed test. **Groups are significantly different at the 0.05 level, two-tailed test. ***Groups are significantly different at the 0.01 level, two-tailed test.

[A] Average proportion of test items answered correctly.

[B] Average standardized test scores (mean = 0. standard deviation = 1).

Table 12.
Intent-to-Treat Impact Estimates—Effects of Grade 4 Exposure to Math Pathways & Pitfalls on Grade 5 Student Outcomes for Latinos and by English Learner Status (Two Years vs. No Exposure)

Grade 5 Measure	Treatment: Two years of MPP[a]	Delayed-treatment control: No MPP[b]	Difference	*p*	Effect size (Diff/SD)
Mathematics Assessment[A]					
Latinos	0.56	0.41	0.15***	.00	0.65
English Learners	0.53	0.38	0.15***	.00	0.72
English-Proficient Students	0.61	0.46	0.15***	.00	0.58
Mathematics Standardized Tests[B]					
Total Mathematics					
Latinos	0.11	-0.12	0.24**	.03	0.25
English Learners	0.01	-0.29	0.30**	.02	0.34
English-Proficient Students	0.31	0.04	0.27**	.05	0.25
Number Sense & Operations					
Latinos	0.14	-.17	0.30***	.01	0.32
English Learners	0.04	-0.28	0.33**	.02	0.37
English-Proficient Students	0.30	-0.06	0.37***	.01	0.37
Mathematical Language Assessment[A]					
Latinos	0.59	0.46	0.13***	.00	0.65
English Learners	0.55	0.40	0.15***	.00	0.76
English-Proficient Students	0.65	0.53	0.11***	.01	0.53
English Language Proficiency[B]					
Latinos	-0.01	-0.03	0.02	.96	0.02
English Learners	0.01	-0.04	0.04	.87	0.05
Listening					
Latinos	-0.02	0.04	-0.06	.85	-0.07
English Learners	-0.04	0.02	-0.06	.86	-0.06
Speaking					
Latinos	-0.07	-0.01	-0.06	.77	-0.06
English Learners	-0.03	-0.05	0.03	.90	0.03

Note. Data are regression-adjusted using multilevel regression models that include baseline test scores and demographic characteristics as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.
[a] Classroom *n* = 44, Latino/a student *n* = 251, English learner *n* = 179, and English proficient *n* = 117 .
[b] Classroom *n* = 20, Latino/a student *n* = 250, English learner *n* = 126, and English proficient *n* = 83.
*Groups are significantly different at the 0.10 level, two-tailed test. **Groups are significantly different at the 0.05 level, two-tailed test. ***Groups are significantly different at the 0.01 level, two-tailed test.
[A] Average proportion of test items answered correctly.
[B] Average standardized test scores (mean = 0. standard deviation = 1).

Cumulative Impacts – Two Years vs. One Year of Math Pathways & Pitfalls

Tables 13 and 14 show results pertaining to research question 2b, focusing on the cumulative impacts of different amounts of student exposure to *Math Pathways & Pitfalls*. Overall, the results for two years of exposure to *Math Pathways & Pitfalls* versus one year indicate the following:

Grade 5 Mathematics

- Project assessment: Two years of exposure to *Math Pathways & Pitfalls* increases scores on the mathematics assessment, as compared to one year, for the full sample of grade 5 students ($p < .05$, $ES = .28$), Latinos ($p < .05$, $ES = .31$), and English-proficient students ($p < .05$, $ES = .34$). Results are marginally significant for English learners ($p = .10$, $ES = .25$).
- Standardized assessment: Two years of exposure to *Math Pathways & Pitfalls* increases standardized mathematics test scores, as compared to one year, for the full sample of grade 5 students, both for Total Mathematics scores ($p < .01$, $ES = .24$) and the *Number Sense & Operations* subscale ($p < .01$, $ES = .24$). These differences are also significant for Latinos ($p < .01$, $ES = .28$ and .29 for total and subscale scores, respectively), and English learners ($p < .01$, $ES = .35$ and .40 for total and subscale scores, respectively).

Grade 5 Language

- Project assessment: Two years of exposure to *Math Pathways & Pitfalls* increases scores on the mathematical language assessment, as compared to one year, for grade 5 Latino/a students ($p < .05$, $ES = .24$). This difference is marginally significant for the full student sample ($p < .10$, $ES = .19$) and for English-proficient students ($p < .10$, $ES = .28$).
- Standardized assessment: Two years of exposure to *Math Pathways & Pitfalls,* as compared to one year, is not associated with increases in scores on standardized English language proficiency tests for grade 5 Latinos or English learners.

Table 13.
Intent-to-Treat Impact Estimates—Effects of Grade 4 Exposure to Math Pathways & Pitfalls on Grade 5 Student Outcomes for Full Student Sample (Two Years vs. One Year)

Grade 5 Measure	Treatment: Two years of MPP[a]	Treatment: One year of MPP[b]	Difference	*p*	Effect size
Mathematics Assessment[A]	0.58	0.51	0.07**	.02	0.28
Mathematics Standardized Tests[B]					
Total Mathematics	0.15	-0.09	0.25***	.00	0.24
Number Sense & Operations	0.15	-0.08	0.24***	.00	0.24
Mathematical Language Assessment[A]	0.59	0.54	0.04*	.08	0.19
English Language Proficiency[B]	-0.01	0.01	-0.03	.85	-0.03
Listening	-0.02	-0.10	0.08	.76	0.08
Speaking	-0.02	0.06	-0.08	.60	-0.09

Note. Data are regression-adjusted using multilevel regression models that include baseline test scores and demographic characteristics as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.

[a] Classroom n = 44, student n = 296.

[b] Classroom n = 42, student n = 206.

*Groups are significantly different at the 0.10 level, two-tailed test. **Groups are significantly different at the 0.05 level, two-tailed test. ***Groups are significantly different at the 0.01 level, two-tailed test.

[A] Average proportion of test items answered correctly.

[B] Average standardized test scores (mean = 0. standard deviation = 1).

Table 14.
Intent-to-Treat Impact Estimates—Effects of Grade 4 Exposure to Math Pathways & Pitfalls on Grade 5 Student Outcomes for Latinos and by English Learner Status (Two Years vs. One Year)

Grade 5 Measure	Treatment: Two years of MPP[a]	Treatment: One year of MPP[b]	Difference	p	Effect size
Mathematics Assessment[A]					
Latinos	0.58	0.51	0.07**	.02	0.31
English Learners	0.55	0.49	0.06*	.09	0.25
English-Proficient Students	0.63	0.55	0.08**	.03	0.34
Mathematics Standardized Tests[B]					
Total Mathematics					
Latinos	0.12	-0.15	0.27***	.00	0.28
English Learners	0.15	-0.18	0.33***	.00	0.35
English-Proficient Students	0.16	0.04	0.12	.34	0.11
Number Sense & Operations†					
Latinos	0.10	-0.18	0.28***	.00	0.29
English Learners	0.20	-0.18	0.38***	.00	0.40
English-Proficient Students	0.10	0.07	0.03	.82	0.03
Mathematical Language Assessment[A]					
Latinos	0.58	0.53	0.05**	.05	0.24
English Learners	0.56	0.53	0.03	.26	0.15
English-Proficient Students	0.63	0.57	0.06*	.07	0.28
English Language Proficiency[B]					
Latinos	-0.04	0.00	-0.03	.82	-0.04
English Learners	-0.01	0.01	-0.03	.85	-0.03
Listening					
Latinos	-0.02	-0.15	0.13	.60	0.12
English Learners	-0.02	-0.10	0.08	.76	0.08
Speaking					
Latinos	-0.08	0.07	-0.15	.36	-0.16
English Learners	-0.02	0.06	-0.08	.60	-0.09

Note. Data are regression-adjusted using multilevel regression models that include baseline test scores, demographic characteristics, and randomization strata as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.

[a] Classroom n = 44, student n = 179.

[b] Classroom n = 42, student n = 125.

*Groups are significantly different at the 0.10 level, two-tailed test. **Groups are significantly different at the 0.05 level, two-tailed test. ***Groups are significantly different at the 0.01 level, two-tailed test.

[A] Average proportion of test items answered correctly.

[B] Average standardized test scores (mean = 0. standard deviation = 1).

Overview of Results of Impact Study

The major aim of the project was to test the effectiveness of *Math Pathways & Pitfalls* for improving student performance in mathematics and mathematical language development. Findings are summarized in Table 15, which shows results for different doses of *Math Pathways & Pitfalls*, for the full student sample and separately for Latinos, English learners, and English-proficient students, respectively. This table compares outcomes for students who had *Math Pathways & Pitfalls* during one year (15 hours) versus students who had no exposure to the program, as well as cumulative effects for students who had *Math Pathways & Pitfalls* for two years (30 hours) as compared to no exposure, and as compared to one year (15 hours). The table indicates the effect size for each significant difference.

As shown in Table 15, just one year (15 hours) of *Math Pathways & Pitfalls* included significant treatment effects on student mathematics achievement and mathematical language, with grade 5 impact stronger than grade 4, but effects were substantially stronger when students had *Math Pathways & Pitfalls* over two years (30 hours). The cumulative results were significant for the full sample of students as well as in disaggregated analyses for Latinos, English learners, and English-proficient student sub-samples. Two years of *Math Pathways & Pitfalls*, when compared to no exposure to the program, raised standardized mathematics test scores, both Total Mathematics scores and *Number Sense & Operations* subscores, as well as scores on project assessments of mathematics and mathematical language, with effect sizes as high as .65 for Latino/a students and .76 for English learners. In grade 5, one year of the program raised standardized *Number Sense & Operations* subscores, and scores on project assessments of mathematics and mathematical language, albeit with more modest effect sizes (.15–.29).

Summary of mathematics results. Just one year of *Math Pathways & Pitfalls* raised scores on project-administered mathematics tests for the full sample of grade 5 students as well as for grade 5 Latino/a students and English learners. One year of *Math Pathways & Pitfalls* also raised grade 5 students' standardized test scores of mathematics achievement, in particular the *Number Sense & Operations* subscale. Significant increases in standardized test scores were found for the full sample of students, and for English-proficient students. One year of *Math Pathways & Pitfalls* did not raise Latinos or English learners' standardized mathematics test scores in grades 4 or 5.

In grade 4, impact of one year of *Math Pathways & Pitfalls* on student mathematics achievement was limited, with marginally significant increases in English learners' project-administered mathematics test scores (effect size of .17), and no other impact on mathematics scores.

Exposure to *Math Pathways & Pitfalls* over two years, in contrast, raised project-administered as well as standardized mathematics test scores for the full sample of students and for the subsamples of Latinos, English learners, and English-proficient students.

Table 15.
Significant Effect Sizes of Different Doses of Math Pathways & Pitfalls for Full Sample of Students, Latinos, English Learners, and English-Proficient Students

Measure	Full sample	Latino/a students	English learners	English-proficient
Grade 4 – One year vs. no exposure				
Project Mathematics Assessment	-	-	.17†	-
Mathematics Standardized Test–Total Mathematics	-	-	-	-
Mathematics Standardized Test–*Number Sense & Operations*	-	-	-	-
Mathematical Language Assessment	.18	.16†	.23	-
Grade 5 – One year vs. no exposure				
Project Mathematics Assessment	.19	.20	.20	.19
Mathematics Standardized Test–Total Mathematics	-	-	-	-
Mathematics Standardized Test–*Number Sense & Operations*	.18	-	-	.29
Mathematical Language Assessment	.15†	-	-	-
Grade 5 – Two years vs. no exposure				
Project Mathematics Assessment	.64	.65	.72	.58
Mathematics Standardized Test–Total Mathematics	.29	.25	.34	.25
Mathematics Standardized Test–*Number Sense & Operations*	.36	.32	.37	.37
Mathematical Language Assessment	.63	.65	.76	.53
Grade 5 – Two years vs. one year				
Project Mathematics Assessment	.28	.31	.25†	.34
Mathematics Standardized Test–Total Mathematics	.24	.28	.35	-
Mathematics Standardized Test–*Number Sense & Operations*	.24	.29	.40	-
Mathematical Language Assessment	.19†	.24	-	.28†

Note. Results of multilevel regression models that include baseline test scores, demographic characteristics, and randomization strata as covariates. Effect sizes calculated by dividing estimates by the pooled standard deviation of the outcome variable.
†Marginally significant – $p \leq .10$. - Not significant – $p > .10$

Summary of mathematical language results. The full impact of *Math Pathways & Pitfalls* is most evident for grade 5 students exposed to the program for two years, as compared to no exposure; mathematical language scores increased substantially (effect sizes .53–.76) for the full sample of students as well as for the Latino/a, English learner, and English-proficient student sub-samples. One year of *Math Pathways & Pitfalls* had more modest effects, raising grade 4 students' scores on mathematical language, for the full sample of grade 4 students and grade 4 English learners, and had marginally significant effects for grade 4 Latino/a students and the full grade 5 sample.

Regardless of length of exposure, *Math Pathways & Pitfalls* is not associated with increases on standardized English language proficiency tests for Latino/a students or English learners.

Teacher Reports on the Impact of *Math Pathways & Pitfalls*

The end-of-year survey completed by treatment teachers in the spring of 2007 (n = 45) included several questions about how they used *Math Pathways & Pitfalls*, including open-ended questions about the value and impact of *Math Pathways & Pitfalls* for students and teachers. The surveys provide some specific information about what teachers valued about *Math Pathways & Pitfalls*.

Questions on the surveys asked teachers to state what was of *most value* about using *Math Pathways & Pitfalls* lessons, and also to describe something that was important to them about using *Math Pathways & Pitfalls*. Overall, the written responses to both questions were quite similar. Some responses describe positive effects of using *Math Pathways & Pitfalls*, such as "richer student discussions" while other responses focused on specific *Math Pathways & Pitfalls* strategies or approaches, such as the *Discussion Builders* (classroom posters with sentence stems). Almost half of the teachers (48%) wrote about improvements in student verbal participation as an important, positive effect of using *Math Pathways & Pitfalls*. (Note that number of responses listed may exceed number of teachers who responded, because teachers often included more than one idea in their responses.) Researchers reviewed and generated thematic groupings for the responses. Below, descriptors of these categories of teacher response are listed, followed by some examples of full-text of teacher responses. The descriptors listed below come from the key words used by teachers in each thematic grouping.

What about using *Math Pathways & Pitfalls* Lessons was of *most value* to you as a teacher? (n = 45)

Student discussion, sharing, collaborating	13
Discussion Builders, sentence starters	9
Lesson plan, structure of the lesson	7
Exploring and discussing pitfalls	6
Students explain/explore own reasoning	5
Materials, it was everything I needed	5
Drawing pictures, using diagrams	5
Using the number line	2
Approach to math	2
Vocabulary lists	2
Having the script (discussion prompts)	2

Some unique or more complex responses:

Re-teaching a concept, mini lessons, alignment to curriculum, stronger students helped weaker, having students explain their reasoning helped me know where the weaknesses were, students explore multiple ways to solve a problem, the *Discussion Builders* provided a safe scaffold for students to express their thinking, it was different, having the film, modeling how to think through a problem, students' growth toward the end of the lesson, lesson five!

Some comments that stood out:

The bringing together of the children's minds to help each other work through the pitfalls [was of most value to me]. Those children strong in math actually helping those weaker and actually enjoying this.

The discussion component. Actually listening to students discuss multiple ways to solve equations – very engaging for them.

Please write a few sentences about something that was important to you about using *Math Pathways & Pitfalls*: [*n* = 33 responses from 45 teachers]

Empowered shy students, students less anxious	11
Student discussion, sharing, collaborating	6
The concept of/ student awareness of pitfalls	6
Students explain/explore own reasoning	4
Some students upset, had difficulty	4
Drawing pictures, using diagrams very helpful	3
Transfer to other lessons	3
Approach to math	3
Discussion Builders, sentence starters	2
Lesson plan, structure of the lesson	2
How teacher-friendly the materials were	2
Worked well with my textbook/ curriculum	2
The discussions I had with the other treatment teacher	1
More than one strategy can lead to right answer	1
Using the number line	1
Having the script (discussion prompts to use with students)	1

Sample responses:

I learned how to use the overhead projector. I also learned a new approach to math. The students learned how to use the overhead projector. When it came time for math, they were ready and eager to share their solutions to problems.

What I really delighted in was seeing the confidence level and watching them try and not be crushed when wrong, still willing to try again.

There is one student who surprised me. She usually cannot grasp math concepts. She was able to use the put together and take apart strategy with equivalent fractions.

My students loved working with a partner and as I walked around, I heard them discussing and helping each other and having arguments about how this is wrong or that's not how the teacher explained it.

It was important that I reviewed the lessons before teaching them to my students. Some of the concepts were taught differently than when I had learned them.

Difficulties Using Math Pathways & Pitfalls

We also asked teachers what *did not* work for them or their students, and we got a wide range of comments and suggestions in response, listed below in their entirety [*n* = 39 responses from 45 teachers]. Many teachers [23%] found it very challenging to teach their units during a specific time of year or over the course of a specific time span, because it did not mesh well with their school's or district's scheduling, or because they had other obligations. Several others [13%] felt that students did not have the necessary background or conceptual knowledge to participate in the *Math Pathways & Pitfalls* lessons without some additional pre-teaching or preparation.

Fitting the program with school schedule/time	9
Concepts weren't taught before lessons	8
Students did not have background knowledge	5
Lessons took longer than planned for	4
Students had difficulties with it/ didn't understand why pitfalls wrong	3
Difficult for class when students present their ideas	3
Did not mesh well with district curriculum	2
Visual aids/posters difficult for students	2
Material delivery not timely	2
Pacing of the lessons	2
Wanted homework/supplemental materials to go with the lessons	2
English prompts too difficult for students/ particularly English learners	2
Video didn't excite students/ was intimidating	2
Students resisted using *Discussion Builders*	2
Difficult to get students to participate, esp. with early problems	2
Hard to know when students really understood lesson	1
Needed more copies/materials	1
Use booklets rather than loose sheets	1
Difficult to motivate students initially	1
Original copies out of order, hard to follow numbering	1
The binder is confusing	1
Following the script exactly as written	1
Some of the student problems more difficult than example problems	1
Students leaving the class	1
Need more support from trainers/site coordinators	1
Every lesson was a success/ no problems	3

[Any of the 6 blank responses could also have been intended to mean "no problems"]

Sample responses:

The timing should have been EARLIER. When the workshops were going on, we (TEACHERS) forgot how impacted we would be with extra helpings on our plates: tests, tests, tests….

The problem for me was when they shared strategy with neighbor. Often there weren't any strategies between the two or three of them and the solutions/ideas were "way out" of the ballpark.

Impact of Math Pathways & Pitfalls on Students' Attitudes and Learning

In the treatment group (n = 56), most teachers reported that the students liked the *Math Pathways & Pitfalls* lessons (77.1% "agreed" or "strongly agreed" to Likert-like prompts). A majority stated that *Math Pathways & Pitfalls* lessons helped stronger students (75.0%), weaker students (64.3%), and English learners (64.3%); fewer than half of the teachers found the lessons helpful for special needs students (44.7%). When asked in open-ended questions to describe the impact of *Math Pathways & Pitfalls* lessons on students' understanding of fractions and decimals, most teachers described positive effects [79% or 34 responses from 43 teachers]. Specific positive impacts that teachers cited included statements that *Math Pathways & Pitfalls* helped students understand:

What fractions and decimals represent	8
Conversion; relating fractions to decimals	3
Place value	2
The mistakes they typically make	2
Subtraction	1

Other teachers stated that, because of *Math Pathways & Pitfalls*, students were better able to express their thinking and remembered concepts better later, for example:

> (I heard) a lot of "Oh, I see what I'm doing wrong." They can pinpoint where their mistakes are.
>
> They stopped saying "point" and read decimals with correct place value more frequently. They transitioned from decimal to fractions with somewhat of an ease.

We also asked teachers to describe the impact of the *Math Pathways & Pitfalls* lessons on students' beliefs about, or attitudes toward math [n = 38 of 45]. Of those who responded, 74% stated that *Math Pathways & Pitfalls* had a generally positive affect on student attitudes or beliefs.

Positive	28
Mixed; not helpful for some (or many)	8
No change ("my students always loved math")	3
Unsure	4

Sample written responses:

> I think the biggest impact *Math Pathways & Pitfalls* lessons made on the students was finding pitfalls – they are not ashamed of making a mistake and going to the board and putting their wrong problem on the board as they know their peers will help them find the pitfall.
>
> I think they believe now that pitfalls are not just made by them.

Implementation Fidelity Study

Overview

This substudy investigated the implementation fidelity of *Math Pathways & Pitfalls* curricula as enacted in participating teachers' classrooms, compared to the structure and procedures that were intended by program developers. Where discrepancies were reported by teachers, the study provides information about the ways in which teachers modified *Math Pathways & Pitfalls* lessons, and reasons for those modifications. Descriptive data on implementation fidelity informs the interpretation of student test results as well as provides information that may suggest refinements of the program. The research questions addressed were:

1. How true were *Math Pathways & Pitfalls* lessons, as implemented by participating teachers, to the number and sequence of lessons in the program?
2. To what extent did teachers use the structure and procedures of components within individual lessons?

3. What changes, if any, did the teachers make in the number, sequence, or procedures of the lessons? What were the reasons for these modifications?

Number and sequence of lessons. At the end of each school year, all teachers who had implemented *Math Pathways & Pitfalls* in their classrooms that year were asked to complete written survey questions about their use of *Math Pathways & Pitfalls* (Appendices G and I). A set of items on these surveys addressed the question of how many lessons teachers implemented (Table 16). Over 90% of teachers in the treatment group implemented all seven *Math Pathways & Pitfalls* in both Years 1 and 2, and the rest implemented all but one of the lessons. Delayed-treatment teachers who implemented the program for the first time in Year 2 varied more in their use of the lessons, with just over 70% implementing all of the seven lessons.

Table 16.
Percent of Teachers Implementing Different Numbers of Math Pathways & Pitfalls Lessons

Number of lessons used (of 7)	Treatment teachers in Year 1 (*n* = 56)	Treatment teachers in Year 2 (*n* = 31)	Delayed-treatment teachers in Year 2 (*n* = 39)
		Math Pathways & Pitfalls Lessons	
7	91.1	93.5	71.8
6	8.9	6.5	5.1
5	0.0	0.0	20.5
1-4	0.0	0.0	2.6
0	0.0	0.0	0.0
		Mini Lesson 1	
7	83.3	86.2	62.2
6	9.3	6.9	8.1
5	5.6	3.4	16.2
1-4	1.9	0.0	13.5
0	0.0	3.4	0.0
		Mini Lesson 2	
7	72.7	69.0	56.8
6	14.5	13.8	2.7
5	5.5	13.8	18.9
1-4	1.8	0.0	13.5
0	5.5	3.4	8.1

Overall, teachers used fewer of the *Math Pathways & Pitfalls* mini lessons than they did the core lessons (see Table 16), with some teachers in each group doing none of the mini lesson 2s. However, over 90% of the treatment teachers used all or all but one of mini lessons 1 in both years, and over 80% used all or all but one of the mini lesson 2s in both years. The pattern of delayed-treatment control teachers using fewer lessons than the treatment teachers held for the mini lessons as well, with 70% using all or all but one of mini lesson 1s, and almost half skipped two or more of the mini lesson 2s.

Structure and procedures of components within lessons. Other survey questions asked teachers about their use of different components within the lessons (Table 17). Over three quarters of the teachers in both in Years 1 and 2 agreed or strongly agreed that they read the Mathematical Background section of the teaching guide for almost every lesson. Approximately 90% of the treatment teachers in both years, and 80% of the delayed-treatment control teachers, indicated that they used the Getting Started tasks at the beginning of the school year to help their students learn how to use the *Discussion Builders*, and 90% of all groups used most of the questions in the teaching guides to help them conduct class discussions. About 77–80% of the treatment teachers in both years also indicated agreed that their students used the *Discussion Builders* in each lesson, although only about 66% of delayed-treatment control teachers in Year 2 did so. Approximately a quarter of teachers in all groups disagreed with a statement that they mostly made up their own questions to conduct class discussions. Overall, teachers reported a strong pattern of fidelity to the internal structure and procedures of the lessons. These findings were further reported on questions about the extent to which teachers used the lessons and procedures (Table 18).

Table 17.
Percent of Teachers in Years 1 and 2 Who Agreed or Strongly Agreed with Each Statement Regarding Use of Math Pathways & Pitfalls Materials

Statement	Group	Agree	Strongly agree
I read the Mathematical Background section of the teaching guide for almost every lesson. *(Check "strongly agree" if you skipped little or nothing in these sections.)*	Year 1 Treatment	48.2	30.4
	Year 2 Treatment	58.1	19.4
	Year 2 Delayed-treatment	47.4	34.2
I used the Getting Started tasks at the beginning of the school year to help my students learn how to use the *Discussion Builders.*	Year 1 Treatment	64.9	24.6
	Year 2 Treatment	51.6	38.7
	Year 2 Delayed-treatment	41.0	41.0
My students used the *Discussion Builders* in each lesson.	Year 1 Treatment	56.1	21.1
	Year 2 Treatment	61.3	19.4
	Year 2 Delayed-treatment	33.3	33.3
I used most of the questions in the teaching guides to help me conduct the discussions with my class.	Year 1 Treatment	64.9	22.8
	Year 2 Treatment	64.5	25.8
	Year 2 Delayed-treatment	61.5	30.8
I mostly made up my own questions to conduct the discussion with my class, instead of just using those in the teaching guide.	Year 1 Treatment	24.6	0.0
	Year 2 Treatment	25.8	6.5
	Year 2 Delayed-treatment	33.3	0.0

Note. Year 1 treatment group $n = 55$. Year 2 treatment group $n = 31$. Year 2 delayed-treatment control group $n = 39$.

Teacher modifications of lesson structure and procedures. In open-ended survey questions, treatment teachers ($n = 56$) in Year 1 were asked whether and how they made changes to the *Math Pathways & Pitfalls* lessons. When asked what parts of the materials and procedures they sometimes did not use, 51% responded that they used the lessons unchanged. About a quarter of the teachers (27.5%) stated that they did leave out some materials:

4 Mathematical Background sheets (3 because they knew the content, didn't need the math background sheets)
6 Mini lessons
1 Overhead transparencies
1 Clipboard discussion
1 Some of the student worksheets
1 Textbooks used during training
1 Didn't get to all discussion questions

The rest of the teachers (21.6%) responded that they modified some procedures, most often citing a lack of time or simply forgetting some steps. Specific changes mentioned include:

9 I didn't get to everything; I sometimes shortened the lessons/had to speed up to finish.
5 I sometimes took more time/an extra day to clarify students' confusion.
1 Spent less time on academic language
1 Spent less time on Things to Remember (difficult for many students to write and express themselves)

1 Took more time when needed; during lesson 5, students had difficulty with the OK, so I extended the lesson to three days.
1 I used some of my own teacher resources to give them extra practice.
1 Pre-teaching concepts, added more vocabulary building.
1 I often helped students with the first question in Your Turn to help get them started.
1 I allowed students to help create Things to Remember. (Students remembered better if they came up with Things to Remember themselves.)
1 I sometimes did the Things to Remember sooner, right after Pitfall and OK.

5 Didn't always redirect students to *Discussion Builders* (if discussion was going well or in another direction), or didn't use more advanced *Discussion Builders* (students were not comfortable with them (*Discussion Builders*).
1 Did not use exact wording.
1 I messed up the format when I was confused. (I was confused about how to make format work, especially Our Turn.)

Implementation Fidelity Study: Discussion

1. How true were *Math Pathways & Pitfalls* lessons, as implemented by participating teachers, to the number and sequence of lessons in the program?
2. To what extent did teachers use the structure and procedures of components within individual lessons?
3. What changes, if any, did the teachers make in the number, sequence, or procedures of the lessons? What were the reasons for these modifications?

Teachers implementing the intervention were asked to replace 15 hours of their regular mathematics curriculum with seven core *Math Pathways & Pitfalls* lessons selected for their grade level, each with two follow-up mini lessons on the same topic. Teacher survey data on implementation fidelity indicated that in Year 1, over 90% of the treatment teachers did teach the seven core lessons, with only a few teachers missing at most one of the lessons. Over 90% of the treatment teachers in Year 1 also taught six or seven mini lesson 1s, although slightly fewer of the mini lesson 2s. If these self-reported frequencies are accurate, they suggest that a most basic criterion of implementation fidelity was satisfied in Year 1—that the intended number and sequence of lessons be used in treatment classrooms.

In Year 2, however, although the treatment group using *Math Pathways & Pitfalls* for the second time maintained their Year 1 level of use, and they reported continued implementation of the major components within the lessons, the delayed-treatment control group reported more variability in numbers of lessons used, and also in their students' use of one lesson component, *Discussion Builders*. Thus, outcomes for the delayed-treatment control group using *Math Pathways & Pitfalls* for the first time in Year 2 may underestimate the potential impact of the program.

Transfer Study

Overview

Analyses of the student assessment data documented in this report and by prior research (Heller, Curtis, Rabe-Hesketh, & Verboncoeur, 2007) have shown that students of *Math Pathways & Pitfalls* teachers, including English learners, scored higher on mathematics and mathematical language tests. In thinking about why *Math Pathways & Pitfalls* might have these effects, broadly speaking, it could be the result of students spending 15 hours in the *Math Pathways & Pitfalls* lessons. Alternatively, the added mathematics learning could result not just from *Math Pathways & Pitfalls* lesson time but from changes in teachers' other mathematics lessons as well. There is some support for the latter possibility – that non-*Math Pathways & Pitfalls* lessons changed. The purpose of the research described in this section was to investigate whether there is objective, observational evidence that is consistent with these self-reported changes.

Data Collection Procedures

We conducted an exploratory study with eight grade 4 *Math Pathways & Pitfalls* teachers in the San Francisco Bay Area. In order to assess the effects of *Math Pathways & Pitfalls* on non- *Math Pathways & Pitfalls* mathematics instruction, and the extent to which teachers generalized and integrated the structure, procedures, and individual components of the *Math Pathways & Pitfalls* approach, each participating teacher was observed while teaching twice during the year, first at the beginning of the year in October 2006, and then six months later in March or April 2007. Observers took qualitative observation notes using a protocol we developed. Observers also collected audio recordings during the observations and conducted brief interviews with the teachers after each observation.

Immediately after each observation, the observer rated each lesson using a three-point rating scale based on the theoretical framework underlying *Math Pathways & Pitfalls*. The rating scale (see Appendix J) describes Beginning, Developing, and Accomplished instructional environments in each of three areas: Mathematics, Language and Discourse, and Equity:

- The *Mathematics* dimension represents in part the level of mathematical challenge of problems or examples students worked on during the lesson. It also encompasses students' and teachers' reasoning about students' mathematical solutions, including their errors.

- The *Language and Discourse* dimension includes who was doing the talking—the proportion of student to teacher talk—and whether students had opportunities to talk among themselves, not just to the teacher.
- The *Equity* dimension refers to the distribution of opportunities for students with various ethnic, linguistic, and cultural backgrounds to participate in learning experiences. Of concern here are such things as the proportion of students participating and type of engagement, along with the extent to which English learners and identifiably quiet students get involved in activities and discourse with their peers.

In addition, at the end of each year all teachers who implemented the *Math Pathways & Pitfalls* lessons completed written survey questions about impact of their use of the lessons on their teaching of non-*Math Pathways & Pitfalls* lessons.

Transfer Study Findings

Transfer Study Findings: Observed Instructional Practices

Over the six-month period between pre- and post-implementation observations, rubric scores increased by about one scale point on the three-point scale and indicated that teachers' instruction generally developed in all three areas (Table 18). During the fall observation, average scores show classrooms in the beginning-developing range (with means around 1.5 on a three-point scale, see Table 16), and in the developing-advanced range during the spring observation (with mean scores around 2.5 on the same three-point scale, see Table 18). All teachers improved in at least one area, and no teacher scored lower during the second observation than on the first. A non-parametric statistical analysis indicated that mean total scores for the post-*Math Pathways & Pitfalls* implementation data (7.3 out of a possible 9 points) were significantly greater than mean total scores (4.8 out of 9) before *Math Pathways & Pitfalls* was implemented (Wilcoxon $T^+ = 36$, $p < .005$, $n = 8$). Effect sizes for gains in mathematics, language and discourse, and equity were 1.8, 2.0, and 2.0, respectively.

Table 18.
Pre- and Post-Math Pathways & Pitfalls Implementation Classroom Observation Scores

	Observations		Math		Language		Equity		Total score	
ID#	Pre Date	Post Date	Pre	Post	Pre	Post	Pre	Post	Pre	Post
01	10/26/06	3/27/07	1	2	1	2	1	2	3	6
02	10/24/06	4/18/07	2	2	2	2	2	3	6	7
03	10/24/06	4/17/07	2	3	2	3	2	3	6	9
05	10/17/06	4/20/07	1	1	2	2	1	2	4	5
07	10/16/06	4/26/07	2	3	2	3	2	3	6	9
08	10/19/06	4/23/07	1	2	1	2	2	2	4	6
09	10/23/06	3/30/07	2	3	2	3	2	3	6	9
10	10/23/06	3/30/07	1	2	1	3	1	2	3	7
		Mean	1.4	2.3	1.5	2.5	1.5	2.5	4.8	7.3
		SD	*0.5*	*0.7*	*0.5*	*0.5*	*0.5*	*0.5*	*1.4*	*1.6*

Note. Rubric scores were on a scale of Beginning (1), Developing (2), and Advanced (3). Effect sizes for gains: Math .9/.5 = 1.8, Language 1/.5 = 2.0, Equity 1/.5 = 2.0.

The following sections describe some of the typical observed changes in each area: mathematics, language and discourse, and equity.

Mathematics

The mathematics section of the rubric asked the observer to rate the lesson in terms of:

- The types of problems or examples students worked on during the lesson
- The quality and level of student participation
- Student willingness to reveal confusion or discuss mistakes
- Teacher response to student errors
- The progress that students made toward understanding the mathematical topic of the lesson.

Data from the post-implementation observations indicated that teachers generally focused on more challenging and difficult problems with their students than they had in pre-implementation observed class meetings. For example, rather than teacher-directed whole class question-and-answer periods, students were observed talking to make sense of the mathematics by sharing, showing, or explaining their ideas to each other. Students were more likely to notice a pitfall and point it out to the class (for example, in a class comparing positive and negative integers, one student pointed out to the others, “we’re not just looking at the digits.” [T03]). Teachers were more likely to view errors positively as an opportunity to learn. And overall, students emerged from the spring lessons with a deeper understanding of the mathematical topics of the lesson.

Language and Discourse

The language and discourse section of the rubric asked the observer to rate the lesson in terms of:

- The relative proportion of student to teacher talk
- The purpose of classroom talk (e.g., procedural vs. exploratory)
- The primary setting of classroom talk (whole class or small group)
- The method of introducing new mathematical vocabulary (as a list or with opportunities to discuss and practice) and confidence with which students use it
- The level of student engagement.

On average, teachers moved away from teacher-dominated whole group discussion, and were observed varying participant structures, and giving students more chances to take on a greater variety of speaker roles (for example, several teachers were observed to give students much more time to think through, work out, and discuss problems in groups, giving them responsibility for recording and analyzing their findings [T 03, 07, 09, 10]). Students were expected to do more than give short answers, and began to explain and justify their reasoning, at times participating in extended discussions with fellow students about mathematical ideas. Students were more likely to use mathematical vocabulary with confidence and showed a greater level of engagement in the lesson over all. (For example, during a pre-intervention lesson, one teacher's expectations for students' use of mathematics vocabulary students seemed to be very low. Most of class was used for lecture and whole group question and answer. During her post-intervention lesson, she made an effort to use a new term (*y*-axis) in varied ways, and students, working with partners, were observed using words such as negative, positive, axis, opposite and units, in their work with partners [T10]).

Equity

The equity section of the rubric asked the observer to rate the lesson in terms of:

- The variety and number of students participating in the lesson
- The respect and attentiveness that students show each other when speaking
- The quality and quantity of participation by English learners and differently-abled students
- The patience, encouragement and wait time demonstrated by the teacher.

In general, a greater variety and percentage of students were participating in group and whole-class discussions during the second observation. Students were more likely to be patient with each other by making positive comments and working together cooperatively. Teachers and students gave each other time to think before jumping in with their own ideas. Some teachers with beginning English learners allowed them to work with partners who were fluent in their language during the first lesson. During the second lesson these teachers successfully encouraged them to participate in whole-class activities, and cited their contributions (for example, T09).

Transfer Study Findings: Teacher Interviews

Participating teachers were interviewed twice, usually around the times their classrooms were observed, in the fall and in the spring. Although the first interview occurred after teachers attended the first or second *Math Pathways & Pitfalls* professional development meeting, we expected to see change in non-*Math Pathways and Pitfalls* mathematics instruction as teachers mastered the *Math Pathways & Pitfalls* lessons and became more familiar with the strategies entailed in the *Math Pathways & Pitfalls* approach.

A comparison of the nature of teacher comments in first and second interviews, revealed that teaching goals remained fairly constant. Teachers continued to want their students to participate more actively, particularly their English learners, though this may have been influenced by how the question was framed (see below for interview questions and summary of responses). However, when asked what they would do to achieve these goals, teachers were much more specific and varied in the strategies and practices they described during the second interview. While most teachers claimed during both interviews that "students were participating more and more" (and it may be generally the case that, as the year progresses, students feel more comfortable participating in their classes), teachers seemed more aware of what might be fostering increased participation during the second interview, and were generally more specific about potential connections between activities and effects.

During the second interview, the interviewer also asked specifically how the *Math Pathways & Pitfalls* intervention was affecting each teacher's non-*Math Pathways & Pitfalls* lessons. In response, teachers cited a wide range of *Math Pathways & Pitfalls* teaching strategies and practices, and they were generally very positive about their *Math Pathways & Pitfalls* experience. In what follows, we give each interview prompt and a summary of teacher responses.

What are your goals for student participation?

More participation especially from weaker students and English learners:
First interview: [4 of 6 teachers]
Second interview: [6 of 6 teachers]

More advanced language use, more math words;
First: [2 of 6 teachers]
Second: [2 of 6 teachers]

Other goals stated during the first interview:
Students explain their thinking [2 of 6 teachers]
Have students use *Math Pathways & Pitfalls* questions [1 of 6 teachers]
Understand what prevents students from participating [1 of 6 teachers]
Have students work as peer tutors, lead mini lessons [1 of 6 teachers]

Other goals stated during second interview:
For students to be less excitable, more on task [1 of 6 teachers]

What have you done to reach your goals?

Had students work with partners or in small groups
First interview: [2 of 6 teachers]
Second interview: [5 of 6 teachers]

Encourage students to talk and explain more, elicit student reasoning:

First interview: [1 of 6 teachers]
Second interview: [2 of 6 teachers]

Let students discuss, figure it out, rather than give answers:
First interview: [0 of 6 teachers]
Second interview: [3 of 6 teachers]

Other strategies and practices described in first interview:
Differentiation into groups [1 of 6 teachers]
Do a daily problem with a diagram [1 of 6 teachers]
Call on students who don't volunteer [1 of 6 teachers]

Other strategies and practices described in second interview:
Encourage students to use sentence frames [3 of 6 teachers]
Have students come up to the board and teach [2 of 6 teachers]
Hear ideas, foster student thinking [2 of 6 teachers]
Use the sticks [2 of 6 teachers]
Use *Math Pathways & Pitfalls* format [1 of 6 teachers]
Use white boards [1 of 6 teachers]
Provide students with multiple ways of responding [1 of 6 teachers]
Use language consciously for English learners' sake [1 of 6 teachers]
Encourage students to help each other [1 of 6 teachers]
Use a karaoke machine, so students speak up [1 of 6 teachers]
Compare English and Spanish math words [1 of 6 teachers]
Building trust [1 of 6 teachers]

How has participation changed since the beginning of the year?

More students are participating:
First interview: [3 of 6 teachers]
Second interview: [6 of 6 teachers]

English learners seem to be more comfortable and confident:
First interview: [4 of 6 teachers]
Second interview: [6 of 6 teachers]

Other changes described in first interview:
Students have figured out who will volunteer, who are the stronger students [2 of 6 teachers]
Students are more motivated about algebra [1 of 6 teachers]
We've built community [1 of 6 teachers]

Other changes described in second interview:
Students use more math vocabulary [1 of 6 teachers]
Think pair share has helped students answer more fully, more articulately [1 of 6 teachers]

In the second interview, over half the teachers stated that participation is still not what he or she would like it to be. [4 of 6 teachers]

In what ways has Math Pathways & Pitfalls transferred to regular math lessons?

- Sentence starters and *Discussion Builders*, and scripts helped students be more structured in their conversations [4 of 6 teachers]
- Teachers and/or students talk about pitfalls [3 of 6 teachers]
- Teachers give students more time "to work out what it means"—really slowing down [3 of 6 teachers]
- I am more deliberate in encouraging participation, watch for it more, and kids know what it sounds like [1 of 6 teachers]
- Having students come up to the board to teach [1 of 6 teachers]
- Partner work [1 of 6 teachers]
- Use a lot more drawing [1 of 6 teachers]

Transfer Study Findings: Surveys

The surveys given to all teachers participating in the treatment group for the impact study in Year 1 contained questions related to the transfer of *Math Pathways & Pitfalls* practices to non-*Math Pathways & Pitfalls* lessons. All teachers (*n* = 56) gave ratings indicating that *Math Pathways & Pitfalls* affected their teaching of regular mathematics lessons to some extent. Almost a quarter of the teachers (23.2%) said that *Math Pathways & Pitfalls* "greatly affected" their teaching of regular mathematics lessons, and over three quarters (76.8%) said that *Math Pathways & Pitfalls* affected their teaching of mathematics "slightly" or "somewhat".

The concept of pitfalls seemed to be powerful for students as well as teachers. Teachers stated that it was useful to identify and discuss pitfalls with students in their regular mathematics lessons, and several teachers said that students would identify pitfalls on their own, after the *Math Pathways & Pitfalls* lessons.

When asked to describe the impact in detail, teachers identified specific *Math Pathways & Pitfalls* strategies or features they saw as evidence of impact [35 responses from 45 teachers].

Discuss/identify pitfalls	10
More discussion in math, richer discussion	7
I welcome multiple strategies to solve problems now	4
I introduce vocabulary, have vocabulary on board	4
I used discussion builders/ sentence starters	3
I have students demonstrating and explaining more	2
I let them work with partners more	2
I design lessons like *Math Pathways & Pitfalls*	2
I use a question to introduce new material	2
I have students draw pictures to help them think	1

Sample responses:

I now think about pitfalls students might make during regular mathematics instruction.

We had more discussions on other ways students were solving problems besides what was taught.

Transfer to lessons in other subjects. Math Pathways & Pitfalls had less of an impact on non-mathematics lessons. While 21% said on survey rating questions that *Math Pathways & Pitfalls* affected their teaching in other subjects "somewhat" or "greatly," 62.5% stated that *Math Pathways & Pitfalls* affected their teaching "not at all" or "slightly" [n = 56]. When asked to write descriptions of the impact of *Math Pathways & Pitfalls* on other lessons [24 responses from 45 teachers], several respondents said they found the use of the *Discussion Builders* very helpful, particularly in subjects like language arts and science. Other teachers said they learned from *Math Pathways & Pitfalls* to allow for more student discussion in other subjects.

How much impact did your experience with Math Pathways & Pitfalls lessons have on your teaching of lessons in other (non-math) subjects?

We used *Discussion Builders* in other lessons	13
I've allowed for more student discussion	5
Focusing on alternative solutions	2
Allowing other students to come up and teach	1
I make more time for questions	1
Put the visual aid on the board	1
More cooperative sharing, learning and tutoring	1
Used "my turn, our turn, your turn"	1
Pitfalls connected to scientific hypothesizing	1

Sample response:

> I tried to incorporate *Discussion Builders* during reading, science, and other subjects in order to get my students into the habit of listening to and questioning items presented to them.

Transfer Study: Discussion

Observations. Over a six-month period between observations of *Math Pathways & Pitfalls* teachers' regular (non-*Math Pathways & Pitfalls*) mathematics lessons, rubric scores increased, indicating that teachers' instruction generally developed in three key dimensions of *Math Pathways & Pitfalls*—mathematics, language, and equity. Mean total scores for post-*Math Pathways & Pitfalls* implementation (7.3 of possible 9 points) were significantly greater than mean total scores (4.8 out of 9) before *Math Pathways & Pitfalls* (Wilcoxon $T^+ = 36$, $p < .005$, $n = 8$). Mean scores for each dimension separately increased from between beginning and developing (1.4, 1.5, 1.5) to between developing and advanced (2.3, 2.5, 2.5). All teachers improved in at least one area, and no teacher scored lower during the second observation than he/she did on the first. The increases in ratings indicate that teaching practices during regular, non-*Math Pathways & Pitfalls* mathematics lessons developed in the direction of the major aspects of *Math Pathways & Pitfalls*' theoretical framework.

Interviews and surveys. Teachers were able to articulate the causes and effects of *Math Pathways & Pitfalls* participation during the second interview, and they were generally more specific about effective strategies they had adopted. Teachers also cited a variety of ways in which *Math Pathways & Pitfalls* affected their non-*Math Pathways & Pitfalls* lessons, and were generally very positive about their *Math Pathways & Pitfalls* experience.

It is interesting to note that a majority of the *Math Pathways & Pitfalls* strategies and improvements that teachers cite are language-related rather than specific to math. This may be due to the large number of English learners in many of these teachers' classrooms. Half the teachers [3 of 6 teachers] noted that some *Math Pathways & Pitfalls* language strategies are effective in all subjects, and are therefore more valuable. Teachers stressed the importance of student talk in math, how much they learned through discussion. Other strategies that teachers associated positively with *Math Pathways & Pitfalls* were related to roles and participant structures (think-pair-share, having students come to the board and teach). Several teachers stated that *Math Pathways & Pitfalls* taught them to make an even more fundamental shift in their role during math classes, "backing off" and "slowing down." They found that when they focused more on what the students had to say, and gave students more time to discuss their solutions and the reasoning behind them, students learned more and became more confident and articulate.

> Because of *Math Pathways & Pitfalls* I'm trying to give the kids time to think and then back off. I don't interrupt them, explain it, finish it up, do more for them. I stop and let the kids figure things out.
>
> Participating in *Math Pathways & Pitfalls* gives us a chance to focus on math in a different way. It inspires kids to do better.
>
> Really, what I was saying before, the importance of language and math, the whole partner work. ... I find myself really slowing down in many respects – (and understanding) the importance of processing. In the past I might have tried to wrap it up today and sort of tell them. *Math Pathways & Pitfalls* is more of a discovery – leads to more time, and having them working through what it means.

In short, these results indicate that teaching practices during regular, non-*Math Pathways & Pitfalls* math lessons developed in the direction of general and specific aspects of the *Math Pathways & Pitfalls* approach that was implemented over the intervening six-month period. This is an important finding because it raises the possibility that classroom practices foundational to *Math Pathways & Pitfalls* lessons may transfer spontaneously to non-*Math Pathways & Pitfalls* lessons. If so, studies of this phenomenon may lead to understandings about which practices are more likely to transfer, and provide clues about how *Math Pathways & Pitfalls* supports the adoption of new practices. Ultimately this information will be valuable in the design of new instructional materials and professional development for teachers.

Summary of Results and Discussion

The purpose of this research was to evaluate the efficacy of *Math Pathways & Pitfalls*, a supplementary curriculum for students with professional development for teachers. Using a cluster-randomized experimental design, the study rigorously examined the experimental effects of teachers using these instructional materials and procedures in place of 15 hours of regular math lessons during each of two academic years. In particular, this study examined the impact of *Math Pathways & Pitfalls* on mathematics achievement and mathematical language development, with special interest in learning more about the effects for Latino/a students and English learners.

Study participants included 126 fourth and fifth grade teachers, and over 3,300 of their consenting students, in Arizona, California, and Illinois. Nearly 70% of the participating students were Latino/a, 55% were classified as English learners, and 75% were eligible for free or reduced-price meals. A volunteer sample of teachers was randomly assigned to experimental and delayed-treatment control groups within schools. Study outcome measures included state standardized mathematics test scores and state standardized English language proficiency test scores, as well as project-administered mathematics assessments, mathematical language assessments. The study was designed to address two primary research questions, one to examine the impact of replacing 15 hours of regular mathematics lessons with *Math Pathways & Pitfalls* lessons, as compared to results for teachers and students using only the regular curriculum, and the second to examine the cumulative impact on students of more than one year of *Math Pathways & Pitfalls* lessons (30 hours).

Results showed that just one year (15 hours) of *Math Pathways & Pitfalls* had significant treatment effects, with grade 5 impact stronger than grade 4, but that effects were substantially stronger when students had *Math Pathways & Pitfalls* over two years (30 hours). *Exposure to Math Pathways & Pitfalls over two years raised project-administered as well as standardized mathematics test scores for the full sample of students and for the sub-samples of Latino/a students, English learners, and English-proficient students.* With respect to language outcomes, for the full sample of students, one year of *Math Pathways & Pitfalls* was marginally associated with increases in scores, while after two years of *Math Pathways & Pitfalls*, the program significantly increased students' mathematical language scores for the full sample and for Latino/a students, English learners, and English-proficient students.

The current results show limited impact of *Math Pathways & Pitfalls* at grade 4, whereas in a National Science Foundation study conducted previously (Heller, Curtis, Rabe-Hesketh, & Verboncoeur, 2007), one year of the program at grade 4 did significantly raise scores on project-administered mathematics assessments. This discrepancy may reflect the fact that the earlier study concentrated the program's 15 hours of focus on one mathematics topic (fractions), whereas here two topics were addressed per grade (fractions and decimals). This meant there were as few as two or three *Math Pathways & Pitfalls* grade 4 lessons on each topic, which may have been too little time for students in that grade. Also, grade 4 students likely had a less developed foundation of fractions and decimals from their regular mathematics studies during prior grade levels than grade 5 students because of the content traditionally covered in those grades. However, in the current study, when students experienced *Math Pathways & Pitfalls* focusing on two topics over two years, that is, in both grades 4 and 5, the program had very powerful cumulative effects.

Teacher survey data on implementation fidelity indicated that in Year 1, over 90% of the treatment teachers did teach the seven core lessons, with only a few teachers missing at most one of the core lessons or any of the mini lessons. If these self-reported frequencies are accurate, they suggest that a most basic criterion of implementation fidelity was satisfied in Year 1—that the intended number and sequence of lessons be used in treatment classrooms.

In Year 2, however, although the treatment group using *Math Pathways & Pitfalls* for the second time reported maintaining their Year 1 level of use and continued implementation of the major components within the lessons, the delayed-treatment control group reported more variability in numbers of lessons used, and also in at least one lesson component. Thus, outcomes for the delayed-treatment control group using *Math Pathways & Pitfalls* for the first time in Year 2 may underestimate the potential impact of the program.

The primary conclusions of this study, stated below, are discussed in relationship to the goals and design elements of the *Math Pathways & Pitfalls* curriculum.

Math Pathways & Pitfalls increases mathematics achievement.

Numerous research studies and mathematics assessments document the stubborn nature of the errors and misconceptions ("pitfalls") students have related to particular mathematics concepts. A unique approach used by *Math Pathways & Pitfalls* to address difficult mathematical concepts is to invite students to compare the correct and flawed (pitfall) solution processes portrayed by the dialogue of two fictional students. Recent research (Durkin & Rittle-Johnson, under review) indicates that the practice of discussing both correct and marked incorrect solution processes leads students to use more correct procedures and remember more correct concepts than students who only compare correct examples. The analysis of correct and incorrect (pitfalls) solutions may not only help students develop correct ways of thinking about mathematical concepts, but also help them work through misconceptions and avoid or detect errors in their work and on assessments. The positive impact on mathematics achievement shown in the current study suggests that *Math Pathways & Pitfalls* helps students develop strong mathematical understanding, that doesn't cave in to pitfalls.

Math Pathways & Pitfalls also emphasizes developing student's metacognition (self-monitoring or awareness of one's own thinking and understanding), which is associated with increased learning (Donovan & Bransford, 2005; White, Frederiksen, & Collins, 2009). The *Math Pathways & Pitfalls* teaching guide suggests prompts to stimulate metacognitive thinking when discussing the narratives of fictitious students' thinking on the student lesson page: "What might Tony have meant when he said…?" "Why so you think Eva decided to divide?" Also, the lessons provide explicit suggestions to instill a propensity for students to "monitor their thinking for pitfalls." Razfar and Leavitt (2010) observed that in *Math Pathways & Pitfalls* classrooms, one of the most dynamic interactions that emerged was what they refer to as the *pitfall metadiscourse.* The *pitfall metadiscourse* allows learners to develop meta-level thinking about a significant feature of learning, which is making mistakes and errors. Through the pitfall narrative, the lesson activity reframes errors as normal and expected, and learners build upon them as opposed to the less inviting 'fixing' stance. The imaginary pitfall narrative in conjunction with the more open discourse structure of *Math Pathways & Pitfalls* encourages students to explore a wider range of problem solving strategies and present them in the public space of the classroom without fear of stigmatization because they might not "doing it the right way."

In brief, the results of the current study indicate that the innovative lesson design of *Math Pathways & Pitfalls* may be especially compelling for developing mathematical understanding and metacognitive awareness of pitfalls.

Math Pathways & Pitfalls has a positive impact on mathematical language development.

Math Pathways & Pitfalls was intentionally designed to support the simultaneous development of mathematical concepts and language, and the power of this dual focus is reflected in findings of this study. The lessons and tools that are part of *Math Pathways & Pitfalls* help students express and comprehend mathematical ideas symbolically and verbally, as well as orally and in writing. Language support is built into numerous aspects of the program:

- all lessons engage students in writing about and discussing mathematics;
- all lessons incorporate the use of varied participation structures, including individual, paired, and whole-group activities, that gradually encourage language use in increasingly risky contexts;
- Math Words are briefly introduced in context to set the stage for discussion;
- *Discussion Builders* provide scaffolding in the form of sentence stems to foster discussion, and *Discussion Builder* posters facilitate students' use of this tool;
- Language Support sections in the teaching guide point out language issues particular to each lesson; and
- narratives of student thinking (in text) on the student's lesson pages provide fodder for discussion

Significant increases on the project-designed test confirm a positive influence of *Math Pathways & Pitfalls* on mathematical language development, which included assessment along five dimensions: *symbolic*, *lexical*, *analytical*, *visual*, and *register* (Solano-Flores, 2010). However, because of the critical role language plays in learning, we also propose that the increases in mathematics achievement shown in this study may be partly attributed to the increased development of mathematical language. The emphasis on mathematical language development in *Math Pathways & Pitfalls* may be particularly helpful for English learners, many of whom were Latino/a in the current study.

Math Pathways & Pitfalls raises standardized test scores for both English learners and English-proficient students, as well as Latino/a students.

Findings in this study showing increases in Latinos' and English learners' standardized mathematics test scores are especially notable since few studies have reported successful interventions for raising the mathematics achievement of these student groups (Khisty, 2002). Studies of classroom discourse during *Math Pathways & Pitfalls* lessons revealed several key elements of effective instruction for Latina/o English and bilingual learners (Khisty & Radosavljević, 2010; Razfar & Leavitt, 2010). Key among these was the observation that students were continuously directed to respond to each others' ideas, and they had models that could be used to scaffold their practice in public discussion. Also, the emphasis on analyzing ways to think about a problem "enables students to have a 'voice' and to have what they say valued by others: this is 'agency.' For bilingual Latinos, agency is a critical element since so much of what happens in schools defines them in terms of deficits and non-agentive behaviors" (Khisty & Radosavljević, 2010, p. 22). Furthermore, Razfar and Leavitt observed that in *Math Pathways & Pitfalls* classrooms, "The positive orientation toward pitfalls not only enhances metacognitive awareness about problem-solving strategies, but it potentially encourages a type of risk-taking practice that is critical to learning and language development" (Razfar & Leavitt, 2010, p. 29).

Math Pathways & Pitfalls embeds multiple ways to support students as they learn to communicate and comprehend mathematical ideas orally and in writing. Tools such as the *Discussion Builders* poster with sentence stems and a video for students model ways to scaffold participation in discussion. The varied participant structures are designed to increase participation among students, especially those who may not yet be comfortable enough with

English to speak in a whole group discussion, or feel they have lower status than other classmates. Students are encouraged to use the key mathematical terms that are introduced at the beginning of the lesson, which helps develop English learners' academic vocabulary and speaking skills during the lesson discussion. The section of the teaching guide that provides mathematical language support specific to that lesson provides additional suggestions for providing greater access for English learners.

Exploratory evidence suggests that key classroom practices may transfer from *Math Pathways & Pitfalls* lessons to regular mathematics lessons.

Math Pathways & Pitfalls lessons were designed to increase the likelihood of key classroom practices from *Math Pathways & Pitfalls* being used in the context of regular mathematics lessons. Lessons intentionally include particular classroom practices that during the program development phase appeared to have the greatest potential for (a) adoption by teachers and students (i.e., feedback from teachers indicated that they believed the benefits of a particular practice advocated by *Math Pathways & Pitfalls* outweighed the drawbacks), (b) a positive impact on student achievement (early evaluations, alignment with standards, focus on common pitfalls), and (c) equitable learning outcomes—practices that align with current research on best practices for English learners.

Tools and structures were developed to assist teachers and students in applying these classroom practices. These classroom practices are adaptable to any mathematics lesson, not just *Math Pathways & Pitfalls*. Results of the Transfer Study suggest that many of the classroom practices emphasized by *Math Pathways & Pitfalls* were spontaneously applied to regular mathematics lessons (non- *Math Pathways & Pitfalls* lessons), particularly the use of *Discussion Builders* sentence stems to support student discussion, identification and discussion about pitfalls, and slowing down to give students a chance to think, write, draw, and talk about their ideas. Results of both the Transfer Study and the randomized controlled experiment raise the possibility that classroom practices foundational to *Math Pathways & Pitfalls* lessons may transfer readily to non- *Math Pathways & Pitfalls* lessons, thus amplifying their impact on learning. Additional research is needed to validate this possibility.

In conclusion, both quantitative and qualitative evidence suggest that *Math Pathways & Pitfalls* lessons produce powerful effects, especially considering the relatively small investment of classroom time (approximately 15 hours per academic year) and professional development support (22-hours during the summer plus 8 hours during the school year).The cumulative effects of longer exposure to the program (30 hours over two years) are substantial for all students, including Latinos and English learners, as well as English-proficient students. If future research and educational programs are to benefit from these findings, we must understand the processes by which *Math Pathways & Pitfalls* influences teaching and supports the adoption of new practices. Ultimately this information can be valuable in the design of effective curricula and instruction for students, and professional development for teachers.

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Appendices available upon request.