Math Pathways & Pitfalls

Standards Alignment Guide

Identifying Math Pathways & Pitfalls Lessons to Support Understanding of the **Common Core State Standards for Mathematics**

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Contents

Background

The *Math Pathways & Pitfalls (MPP)* K–8 supplemental curriculum is designed according to a vision of rigorous and equitable academic experiences and personal agency enacted through Five Foundational Principles for how children learn mathematics:

- Building Mathematical Discussions
- Making Sense
- Confronting Pitfalls
- Visualizing and Connecting
- Capturing Key Ideas

These principles are informed by research findings and classroom instruction practices and align with the National Council of Teachers of Mathematics publications on effective, equitable mathematics teaching practices, such as *Principles to Actions*, the *Taking Action* series, and the *Catalyzing Change* series (NCTM, 2014, 2017a, 2017b, 2020a, 2020b).

The *MPP* units focus on the following mathematics content for grades K–3: place value, whole number sense, and operations with whole numbers. The focus for grades 4–8 is rational number concepts and operations: fractions, decimals, percents, and ratios and proportions. And algebraic reasoning is woven throughout all units.

Purpose

One of the most critical goals of teaching and learning mathematics is to provide multiple experiences of grade-level content standards and practices so students can gain a deep understanding of the mathematics that will reinforce previous learning and prepare them for the next grade or course. Because *MPP* is a supplemental curriculum, the intent of this alignment guide is to help educators determine which *MPP* lessons align with and complement their main curriculum so they can meet this important goal, without requiring a large investment of planning time.

How to Navigate This Document

This standards alignment can be used in two different ways:

1. **Standards ▶ Lessons:** Use the tables that have the standards in the left column to find *MPP* lessons that align with specific standards. This approach can be useful for determining how to further engage students around a specific topic or skill, whether for intervention or enrichment.
2. **Lessons ▶ Standards:** Use the tables that have the *MPP* lessons in the left column to determine which standards are addressed within each *MPP* lesson. This approach can be useful for helping students make connections to previous and future learning.

Workshops in Support of This Alignment

Teachers, schools, and districts are encouraged to reference this alignment guide when using the *MPP* lesson books and/or during *MPP* professional development sessions.

Given that some teachers may desire additional support and collaborative practice to align their standards-based lessons with *MPP* lessons, WestEd offers interactive in-person and virtual workshops to assist teachers in using this alignment document and overall implementation of *MPP* lessons. See the *Math Pathways & Pitfalls* website at https://mpp.wested.org/professional-development/ for additional information.

Common Core State Standards for Mathematics (CCSS-M)

Aligned With Math Pathways & Pitfalls Lessons

KINDERGARTEN: Counting and Cardinality (K.CC)

Know number names and the count sequence. (K.CC.A)	***MPP* units and lessons addressing standard**
K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Unit 1: Lessons 1, 2, 4
K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects).	Unit K: Lessons 6, 7, 8

Count to tell the number of objects. (K.CC.B)	***MPP* units and lessons addressing standard**
K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	K.CC.B.4.a Unit K: Lessons 3, 4, 5, 6, 7, 8 K.CC.B.4.b Unit K: Lessons 3, 4, 5, 6, 7, 8 K.CC.B.4.c Unit K: Lessons 4, 5 Unit 1: Lesson 3

Count to tell the number of objects. (K.CC.B)	***MPP* units and lessons addressing standard**
K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	Unit K: Lessons 3, 4, 5, 6, 7, 8, 9, 10

Compare numbers. (K.CC.C)	***MPP* units and lessons addressing standard**
K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching and counting strategies).	Unit K: Lessons 1, 2, 3, 4, 5, 10 Unit 1: Lesson 3

KINDERGARTEN: Operations and Algebraic Thinking (K.OA)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (K.OA.A)	***MPP* units and lessons addressing standard**
K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Unit K: Lessons 8, 9, 10
K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10 (e.g., by using objects or drawings to represent the problem).	Unit K: Lessons 9, 10 Unit 1: Lesson 3

KINDERGARTEN: Number and Operations in Base Ten (K.NBT)

Work with numbers 11 to 19 to gain foundations for place value. (K.NBT.A)	***MPP* units and lessons addressing standard**
K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones (e.g., by using objects or drawings), and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Unit 1: Lesson 7

KINDERGARTEN: Measurement and Data (K.MD)

Describe and compare measurable attributes. (K.MD.A)	***MPP* units and lessons addressing standard**
K.MD.A.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	Unit K: Lessons 1, 2
K.MD.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter*.	Unit K: Lessons 1, 2 Unit 1: Lesson 3

KINDERGARTEN: Geometry (K.G)

Identify and describe shapes. (K.G.A)	***MPP* units and lessons addressing standard**
K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind*, and *next to*.	Unit K: Lesson 4

GRADE 1: Operations and Algebraic Thinking (1.OA)

Represent and solve problems involving addition and subtraction. (1.OA.A)	***MPP* units and lessons addressing standard**
1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).	Unit 1: Lessons 3, 4, 9

Add and subtract within 20. (1.OA.C)	***MPP* units and lessons addressing standard**
1.OA.C.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	Unit K: Lessons 9, 10 Unit 1: Lessons 1, 2, 4
1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).	Unit K: Lessons 9, 10 Unit 1: Lessons 1, 2, 4, 10 Unit 2: Lesson 4

Work with addition and subtraction equations. (1.OA.D)	***MPP* units and lessons addressing standard**
1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.*	Unit 1: Lesson 6
1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = \square - 3$, $6 + 6 = \square$.*	Unit 1: Lessons 6, 7, 8, 10 Unit 3: Lesson 7

GRADE 1: Number and Operations in Base Ten (1.NBT)

Understand place value. (1.NBT.B)	***MPP* units and lessons addressing standard**
1.NBT.B.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	Unit 1: Lessons 5, 7, 9, 10

Use place value understanding and properties of operations to add and subtract. (1.NBT.C)	***MPP* units and lessons addressing standard**
1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	Unit 1: Lessons 7, 8 Unit 2: Lessons 3, 4 Unit 3: Lesson 7

GRADE 2: Operations and Algebraic Thinking (2.OA)

Represent and solve problems involving addition and subtraction. (2.OA.A)	***MPP* units and lessons addressing standard**
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	Unit 2: Lesson 4

Add and subtract within 20. (2.OA.B)	***MPP* units and lessons addressing standard**
2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.	Unit 2: Lessons 7

GRADE 2: Number and Operations in Base Ten (2.NBT)

Understand place value. (2.NBT.A)	***MPP* units and lessons addressing standard**
2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	Unit 2: Lessons 6, 8 Unit 3: Lessons 8, 9
2.NBT.A.2 Count within 1000; skip count by 5s, 10s, and 100s.	Unit 2: Lessons 3, 6, 8 Unit 3: Lessons 2, 3, 4, 5, 6
2.NBT.A.3 Read and write numbers up to 1000 using base-ten numerals, number names, and expanded form.	Unit 2: Lessons 6, 8 Unit 3: Lessons 6, 8

Use place value understanding and properties of operations to add and subtract. (2.NBT.B)	***MPP* units and lessons addressing standard**
2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Unit 2: Lessons 3, 5, 8, 9, 10 Unit 3: Lesson 3
2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.	Unit 2: Lessons 3, 5, 10

Use place value understanding and properties of operations to add and subtract. (2.NBT.B)	***MPP* units and lessons addressing standard**
2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	Unit 2: Lessons 3, 4, 5, 6, 8, 9, 10 Unit 3: Lessons 3, 4, 5, 9
2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Unit 2: Lesson 6 Unit 3: Lesson 6
2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.	Unit 2: Lessons 3, 4, 5, 6, 8 Unit 3: Lessons 3, 4, 5, 6, 7, 8

GRADE 2: Measurement and Data (2.MD)

Relate addition and subtraction to length. (2.MD.B)	***MPP* units and lessons addressing standard**
2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.	Unit 2: Lessons 1, 2, 3, 4 Unit 3: Lessons 3, 4, 5

GRADE 3: Operations and Algebraic Thinking (3.OA)

Represent and solve problems involving multiplication and division. (3.OA.A)	***MPP* units and lessons addressing standard**
3.OA.A.1 Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each). *For example, describe a context in which a total number of objects can be expressed as 5×7.*	Unit 3: Lessons 10, 11

Represent and solve problems involving multiplication and division. (3.OA.A)	***MPP* units and lessons addressing standard**
3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	Unit 3: Lessons 10, 11 Note: These lessons do not deal with division.
3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*	Unit 3: Lesson 10

Understand properties of multiplication and the relationship between multiplication and division. (3.OA.B)	***MPP* units and lessons addressing standard**
3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Unit 3: Lesson 11

Multiply and divide within 100. (3.OA.C)	***MPP* units and lessons addressing standard**
3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Unit 3: Lessons 10, 11

GRADE 3: Number and Operations in Base Ten (3.NBT)

Use place value understanding and properties of operations to perform multi-digit arithmetic. (3.NBT.A)	***MPP* units and lessons addressing standard**
3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	Unit 3: Lessons 1, 2
3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Unit 2: Lesson 8 Unit 3: Lesson 6

GRADE 3: Number and Operations – Fractions (3.NF)

Develop understanding of fractions as numbers. (3.NF.A)	***MPP* units and lessons addressing standard**
3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	Unit 4: Lessons 1, 2, 6

GRADE 4: Number and Operations in Base Ten (4.NBT)

Use place value understanding and properties of operations to perform multi-digit arithmetic. (4.NBT.B)	***MPP* units and lessons addressing standard**
4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using a standard algorithm.	Unit 2: Lessons 5, 9, 10 Unit 3: Lesson 9
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Unit 3: Lesson 11

GRADE 4: Number and Operations – Fractions (4.NF)

Extend understanding of fraction equivalence and ordering. (4.NF.A)	***MPP* units and lessons addressing standard**
4.NF.A.1 Explain why a fraction *a/b* is equivalent to a fraction *(n x a)/(n x b)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.	Unit 4: Lessons 1, 2, 4, 6, 7, 8
4.NF.A.2 Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	Unit 4: Lessons 1, 4, 6

Build fractions from unit fractions. (4.NF.B)	***MPP* units and lessons addressing standard**
4.NF.B.3 Understand a fraction *a/b* with *a* > 1 as a sum of fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model). *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).	4.NF.B.3.a Unit 4: Lesson 9 4.NF.B.3.b Unit 4: Lesson 3 4.NF.B.3.c Unit 4: Lesson 5

Understand decimal notation for fractions, and compare decimal fractions. (4.NF.C)	***MPP* units and lessons addressing standard**
4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100*.	Unit 5: Lesson 1
4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.	Unit 5: Lessons 1, 2, 4, 5 Unit 6: Lesson 4

GRADE 5: Number and Operations in Base Ten (5.NBT)

Understand the place value system. (5.NBT.A)	***MPP* units and lessons addressing standard**
5.NBT.A.3 - Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form (e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	5.NBT.A.3.a Unit 5: Lessons 3, 6 5.NBT.A.3.b Unit 5: Lesson 6

Perform operations with multi-digit whole numbers and with decimals to hundredths. (5.NBT.B)	***MPP* units and lessons addressing standard**
5.NBT.B.7 - Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Unit 5: Lessons 7, 8, 9, 10, 11

GRADE 5: Number and Operations-Fractions (5.NF)

Use equivalent fractions to add and subtract fractions. (5.NF.A)	***MPP* units and lessons addressing standard**
5.NF.A.1 - Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	Unit 4: Lessons 7, 8
5.NF.A.2 - Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	Unit 4: Lesson 7

Apply and extend previous understandings of multiplication and division. (5.NF.B)	***MPP* units and lessons addressing standard**
5.NF.B.4 - Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) × q* as a parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general,* (a/b) × (c/d) = (ac)/(bd)). b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	5.NF.B.4.a Unit 4: Lessons 10, 11

GRADE 6: Ratio and Proportion (6.RP)

Expectations for unit rates in this grade are limited to non-complex fractions.

Understand ratio concepts and use ratio reasoning to solve problems. (6.RP.A)	***MPP* units and lessons addressing standard**
6.RP.A.1 - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	Unit 7: Lessons 1, 2, 8, 9, 10
6.RP.A.2 - Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	Unit 7: Lessons 1, 5, 6

Understand ratio concepts and use ratio reasoning to solve problems. (6.RP.A)	***MPP* units and lessons addressing standard**
6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	6.RP.A.3 Unit 7: Lessons 1, 2, 7 6.RP.A.3.b Unit 7: Lessons 5, 6 6.RP.A.3.c Unit 6: Lessons 1, 2, 6, 7, 9, 10 6.RP.A.3.d Unit 7: Lesson 11

GRADE 6: The Number System (6.NS)

Compute fluently with multi-digit numbers and find common factors and multiples. (6.NS.B)	***MPP* units and lessons addressing standard**
6.NS.B.2 - Fluently divide multi-digit numbers using a standard algorithm.	Unit 7: Lesson 9
6.NS.B.3 - Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	Unit 5: Lesson 10 Unit 6: Lessons 6, 7, 11 Unit 7: Lesson 6

GRADE 6: Expressions and Equations (6.EE)

Reason about and solve one-variable equations and inequalities. (6.EE.B)	***MPP* units and lessons addressing standard**
6.EE.B.5 - Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Unit 4: Lesson 11 Unit 6: Lessons 8, 9, 10 Unit 7: Lessons 8, 9, 10, 11
6.EE.B.6 - Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	Unit 6: Lessons 8, 9, 10 Unit 7: Lessons 9, 10
6.EE.B.7 - Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.	Unit 6: Lesson 9

GRADE 7: Ratio and Proportion (7.RP)

Analyze proportional relationships and use them to solve real-world and mathematical problems. (7.RP.A)	***MPP* units and lessons addressing standard**
7.RP.A.2 - Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost* t *is proportional to the number* n *of items purchased at a constant price* p*, the relationship between the total cost and the number of items can be expressed as* t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	7.RP.A.2 Unit 7: Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 7.RP.A.2.a Unit 7: Lesson 2 7.RP.A.2.b Unit 7: Lesson 5

Analyze proportional relationships and use them to solve real-world and mathematical problems. (7.RP.A)	***MPP* units and lessons addressing standard**
7.RP.A.3 - Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	Unit 6: Lesson 11

GRADE 7: The Number System (7.NS)

Apply and extend previous understanding of operations with fractions. (7.NS.A)	***MPP* units and lessons addressing standard**
7.NS.A.2 - Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If (p and q are integers, then $–(p/q) = (–p)/q = (p/(–q)$. Interpret quotients of rational numbers by describing real-world context. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	7.NS.A.2.d Unit 6: Lessons 3, 5

Math Pathways & Pitfalls Lessons
Aligned With Common Core State Standards for Mathematics (CCSS-M)

Unit K: Early Math & Number Concepts	CCSS-M Standards
Lesson 1: Which Train Is Longer?	K.CC.C.6 K.MD.A.1 K.MD.A.2
Lesson 2: Which Train Is Shorter?	K.CC.C.6 K.MD.A.1 K.MD.A.2
Lesson 3: Same Number or Not?	K.CC.B.4.a K.CC.B.4.b K.CC.B.5 K.CC.C.6
Lesson 4: Which Row Has More?	K.CC.B.4.a K.CC.B.4.b K.CC.B.4.c K.CC.B.5 K.CC.C.6 K.G.A.1
Lesson 5: Which Row Has Fewer?	K.CC.B.4.a K.CC.B.4.b K.CC.B.4.c K.CC.B.5 K.CC.C.6
Lesson 6: Counting Ladybug Dots	K.CC.A.3 K.CC.B.4.a K.CC.B.4.b K.CC.B.5
Lesson 7: Counting Beads	K.CC.A.3 K.CC.B.4.a K.CC.B.4.b K.CC.B.5

Unit K: Early Math & Number Concepts	CCSS-M Standards
Lesson 8: Count Out 3	K.CC.A.3 K.CC.B.4.a K.CC.B.4.b K.CC.B.5 K.OA.A.1
Lesson 9: Cross Off 4	K.CC.B.5 K.OA.A.1 K.OA.A.2 1.OA.C.5 1.OA.C.6
Lesson 10: Counting Extras	K.CC.B.5 K.CC.C.6 K.OA.A.1 K.OA.A.2 1.OA.C.5 1.OA.C.6

Unit 1: Number, Place Value, & Operations	CCSS-M Standards
Lesson 1: Points on a Number Line	K.CC.A.2 1.OA.C.5 1.OA.C.6
Lesson 2: Counting On a Few	K.CC.A.2 1.OA.C.5 1.OA.C.6
Lesson 3: How Many More?	K.CC.B.4.c K.CC.C.6 K.OA.A.2 K.MD.A.2 1.OA.A.1
Lesson 4: Addition Story Problems	K.CC.A.2 1.OA.A.1 1.OA.C.5 1.OA.C.6
Lesson 5: Tens and Ones Flip	1.NBT.B.2

Unit 1: Number, Place Value, & Operations	CCSS-M Standards
Lesson 6: What Number Is Hiding? (Algebra Readiness)	1.OA.D.7 1.OA.D.8
Lesson 7: Adding to Teen Numbers	K.NBT.A.1 1.OA.D.8 1.NBT.B.2 1.NBT.C.4
Lesson 8: Adding Tens and Ones	1.OA.D.8 1.NBT.C.4
Lesson 9: How Many Are Left?	1.OA.A.1 1.NBT.B.2
Lesson 10: Teen Number Subtraction	1.OA.C.6 1.OA.D.8 1.NBT.B.2

Unit 2: Number, Place Value, & Operations	CCSS-M Standards
Lesson 1: Number Line Points	2.MD.B.6
Lesson 2: Marking Points on a Number Line	2.MD.B.6
Lesson 3: Add a Few	1.NBT.C.4 2.NBT.A.2 2.NBT.B.5 2.NBT.B.6 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 4: What's the Difference?	1.OA.C.6 1.NBT.C.4 2.OA.A.1 2.NBT.B.7 2.NBT.B.9 2.MD.B.6

Unit 2: Number, Place Value, & Operations	CCSS-M Standards
Lesson 5: Don't Squeeze the Digits	2.NBT.B.5 2.NBT.B.6 2.NBT.B.7 2.NBT.B.9 4.NBT.B.4
Lesson 6: Adding On More Tens or Ones	2.NBT.A.1 2.NBT.A.2 2.NBT.A.3 2.NBT.B.7 2.NBT.B.8 2.NBT.B.9
Lesson 7: What Goes in the Blank? (Algebra Readiness)	2.OA.B.2
Lesson 8: Place Value Hints (Algebra Readiness)	2.NBT.A.1 2.NBT.A.2 2.NBT.A.3 2.NBT.B.5 2.NBT.B.7 2.NBT.B.9 3.NBT.A.2
Lesson 9: Minus a Few	2.NBT.B.5 2.NBT.B.7 4.NBT.B.4
Lesson 10: Regroup and Subtract	2.NBT.B.5 2.NBT.B.6 2.NBT.B.7 4.NBT.B.4

Unit 3: Number, Regrouping, & Operations	CCSS-M Standards
Lesson 1: Number Line Sense	3.NBT.A.1
Lesson 2: Marking Points for Numbers	2.NBT.A.2 3.NBT.A.1

Unit 3: Number, Regrouping, & Operations	CCSS-M Standards
Lesson 3: Add On a Bit More	2.NBT.A.2 2.NBT.B.5 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 4: A Little Less	2.NBT.A.2 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 5: Finding the Difference	2.NBT.A.2 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 6: Add On Using Place Value	2.NBT.A.2 2.NBT.A.3 2.NBT.B.8 2.NBT.B.9 3.NBT.A.2
Lesson 7: What Number Is Missing? (Algebra Readiness)	1.OA.D.8 1.NBT.C.4 2.NBT.B.9
Lesson 8: Values of Digits (Algebra Readiness)	2.NBT.A.1 2.NBT.A.3 2.NBT.B.9
Lesson 9: Regroup a Ten	2.NBT.A.1 2.NBT.B.7 4.NBT.B.4
Lesson 10: When Do You Multiply?	3.OA.A.1 3.OA.A.3 (Note: This lesson does not incorporate division.) 3.OA.A.4 3.OA.C.7

Unit 3: Number, Regrouping, & Operations	CCSS-M Standards
Lesson 11: Making Sense and Multiplying	3.OA.A.1 3.OA.A.3 (Note: This lesson does not incorporate division.) 3.OA.B.5 3.OA.C.7 4.NBT.B.5

Unit 4: Fractions	CCSS-M Standards
Lesson 1: Naming Equivalent Fractions	3.NF.A.3 4.NF.A.1 4.NF.A.2
Lesson 2: Fraction Flags	3.NF.A.3 4.NF.A.1
Lesson 3: Fractions on a Number Line	4.NF.B.3.b
Lesson 4: Naming Fractions in Lowest Terms	4.NF.A.1 4.NF.A.2
Lesson 5: Equal Fractions and Mixed Numbers	Supports cluster 4.NF.B.3
Lesson 6: Comparing Fraction Amounts	3.NF.A.3 4.NF.A.1 4.NF.A.2
Lesson 7: Adding Fractions	4.NF.A.1 5.NF.A.1 5.NF.A.2
Lesson 8: Sums More or Less Than 1	4.NF.A.1 5.NF.A.1
Lesson 9: Fraction Subtraction	4.NF.B.3.a
Lesson 10: Multiplying Fractions	5.NF.B.4
Lesson 11: Unknown Fractional Parts of a Number (Algebra Readiness)	5.NF.B.4.a 6.EE.B.5

Unit 5: Decimals	CCSS-M Standards
Lesson 1: Shading Decimal Amounts	4.NF.C.5 4.NF.C.6
Lesson 2: Decimals Are Fractions Too	4.NF.C.6
Lesson 3: Naming and Drawing Decimal Amounts	5.NBT.A.3.a
Lesson 4: Approximating Decimals for Fractions	4.NF.C.6
Lesson 5: Decimals More or Less Than 1	4.NF.C.6
Lesson 6: Comparing and Ordering Decimals	5.NBT.A.3.a 5.NBT.A.3.b
Lesson 7: Finding Sums and Differences Mentally	5.NBT.B.7
Lesson 8: Compute Decimal Sums and Differences	5.NBT.B.7
Lesson 9: Multiplying Decimals	5.NBT.B.7
Lesson 10: Dividing Decimals	5.NBT.B.7 6.NS.B.3
Lesson 11: Unknown Decimal Addends (Algebra Readiness)	5.NBT.B.7

Unit 6: Percents	CCSS-M Standards
Lesson 1: Percent Names for Shaded Areas	6.RP.A.3.c
Lesson 2: Relating Percent and Fraction Amounts	6.RP.A.3.c
Lesson 3: Fractions Into Percents	7.NS.A.2.d
Lesson 4: Percents, Fractions, and Decimals	4.NF.C.6
Lesson 5: Changing Ratios to Percents	7.NS.A.2.d
Lesson 6: Finding Percent Mentally	6.RP.A.3.c 6.NS.B.3
Lesson 7: More or Less Than 10%	6.RP.A.3.c 6.NS.B.3

Unit 6: Percents	CCSS-M Standards
Lesson 8: Translate Into Percent Equations (Algebra Readiness)	6.EE.B.5 6.EE.B.6
Lesson 9: Percent Equations (Algebra Readiness)	6.RP.A.3.c 6.EE.B.5 6.EE.B.6 6.EE.B.7
Lesson 10: Percent Problems and Proportions (Algebra Readiness)	6.RP.A.3.c 6.EE.B.5 6.EE.B.6
Lesson 11: Percent Discount or Markup (Algebra Readiness)	6.NS.B.3 7.RP.A.3

Unit 7: Ratios & Proportions	CCSS-M Standards
Lesson 1: Are the Ratios Equal?	6.RP.A.1 6.RP.A.2 6.RP.A.3 7.RP.A.2
Lesson 2: Making Equal Ratios	6.RP.A.1 6.RP.A.3 7.RP.A.2 7.RP.A.2.a
Lesson 3: 25 Times as Many	7.RP.A.2
Lesson 4: 2.5 Times as Many Jumps	7.RP.A.2
Lesson 5: How Many for 1?	6.RP.A.2 6.RP.A.3.b 7.RP.A.2 7.RP.A.2.b
Lesson 6: Comparing Ratios	6.RP.A.2 6.RP.A.3.b 6.NS.B.3 7.RP.A.2
Lesson 7: Simplify to Find Equivalent Ratios	6.RP.A.3 7.RP.A.2

Unit 7: Ratios & Proportions	CCSS-M Standards
Lesson 8: Solve for *x* in Proportions (Algebra Readiness)	6.RP.A.1 6.EE.B.5 7.RP.A.2
Lesson 9: Setting Up Proportions (Algebra Readiness)	6.RP.A.1 6.NS.B.2 6.EE.B.5 6.EE.B.6 7.RP.A.2
Lesson 10: Similar Figures (Algebra Readiness)	6.RP.A.1 6.EE.B.5 6.EE.B.6 7.RP.A.2
Lesson 11: Proportion Problem or Not? (Algebra Readiness)	6.RP.A.3.d 6.EE.B.5

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