

## Math Pathways & Pitfalls

## Standards Alignment Guide

Identifying Math Pathways & Pitfalls Lessons to Support Understanding of the

**Arizona Mathematics Standards (AMS)** 





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## Background

The *Math Pathways & Pitfalls (MPP)* K–8 supplemental curriculum is designed according to a vision of rigorous and equitable academic experiences and personal agency enacted through Five Foundational Principles for how children learn mathematics:

- » Building Mathematical Discussions
- » Making Sense
- » Confronting Pitfalls
- » Visualizing and Connecting
- » Capturing Key Ideas

These principles are informed by research findings and classroom instruction practices and align with the National Council of Teachers of Mathematics publications on effective, equitable mathematics teaching practices, such as *Principles to Actions*, the *Taking Action* series, and the *Catalyzing Change* series (NCTM, 2014, 2017a, 2017b, 2020a, 2020b).

The *MPP* units focus on the following mathematics content for grades K–3: place value, whole number sense, and operations with whole numbers. The focus for grades 4–8 is rational number concepts and operations: fractions, decimals, percents, and ratios and proportions. And algebraic reasoning is woven throughout all units.

## Purpose

One of the most critical goals of teaching and learning mathematics is to provide multiple experiences of grade-level content standards and practices so students can gain a deep understanding of the mathematics that will reinforce previous learning and prepare them for the next grade or course. Because MPP is a supplemental curriculum, the intent of this alignment guide is to help educators determine which MPP lessons align with and complement their main curriculum so they can meet this important goal, without requiring a large investment of planning time.



## How to Navigate This Document

This standards alignment can be used in two different ways:

- Standards ➤ Lessons: Use the tables that have the standards in the left column to find MPP lessons that align with specific standards. This approach can be useful for determining how to further engage students around a specific topic or skill, whether for intervention or enrichment.
- 2. Lessons ➤ Standards: Use the tables that have the *MPP* lessons in the left column to determine which standards are addressed within each *MPP* lesson. This approach can be useful for helping students make connections to previous and future learning.

# Workshops in Support of This Alignment

Teachers, schools, and districts are encouraged to reference this alignment guide when using the *MPP* lesson books and/or during *MPP* professional development sessions.

Given that some teachers may desire additional support and collaborative practice to align their standards-based lessons with MPP lessons, WestEd offers interactive in-person and virtual workshops to assist teachers in using this alignment document and overall implementation of MPP lessons. See the  $Math\ Pathways\ &\ Pitfalls\$ website at <a href="https://mpp.wested.org/professional-development/">https://mpp.wested.org/professional-development/</a> for additional information.





## **Arizona Mathematics Standards (AMS)**

## Aligned With Math Pathways & Pitfalls Lessons

#### **KINDERGARTEN:** Counting and Cardinality (K.CC)

K.CC.A Know number names and the count sequence.	MPP units and lessons addressing standard
K.CC.A.2  Count forward from a given number other than one, within the known sequence (e.g., "Starting at the number 5, count up to 11").	Unit 1: Lessons 1, 2, 4
K.CC.A.3  Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects).	Unit K: Lessons 6, 7, 8

K.CC.B  Count to tell the number of objects.	MPP units and lessons addressing standard
<ul> <li>K.CC.B.4</li> <li>Understand the relationship between numbers and quantities; connect counting to cardinality.</li> <li>a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one to one correspondence).</li> </ul>	K.CC.B.4.a Unit K: Lessons 3, 4, 5, 6, 7, 8  K.CC.B.4.b Unit K: Lessons 3, 4, 5, 6, 7, 8
<ul> <li>b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted (cardinality).</li> <li>c. Understand that each successive number name refers to a quantity that is one larger (hierarchical inclusion).</li> </ul>	Unit K: Lessons 4, 5 Unit 1: Lesson 3
K.CC.B.5  Count to answer questions about "How many?" when 20 or fewer objects are arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects.	Unit K: Lessons 3, 4, 5, 6, 7, 8, 9, 10



K.CC.C Compare numbers.	MPP units and lessons addressing standard
K.CC.C.6  Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to 10 objects.)	Unit K: Lessons 1, 2, 3, 4, 5, 10 Unit 1: Lesson 3

#### **KINDERGARTEN:** Operations and Algebraic Thinking (OA)

K.OA.A Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	MPP units and lessons addressing standard
K.OA.A.1  Represent addition and subtraction concretely.	Unit K: Lessons 8, 9, 10
K.OA.A.2  Solve addition and subtraction word problems and add and subtract within 10.	Unit K: Lessons 9, 10 Unit 1: Lesson 3

#### **KINDERGARTEN:** Number and Operations in Base Ten (NBT)

K.NBT.A  Work with numbers 11 to 19 to gain foundations for place value.	MPP units and lessons addressing standard
K.NBT.A.1  Compose and decompose numbers from 11 to 19 into 10 ones and additional ones by using objects, drawings, and/or equations. Understand that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones (e.g., 18 = 10 + 8).	Unit 1: Lesson 7



#### **KINDERGARTEN:** Measurement and Data (MD)

K.MD.A  Describe and compare measurable attributes.	MPP units and lessons addressing standard
K.MD.A.1  Describe measurable attributes of a single object (e.g., length and weight).	Unit K: Lessons 1, 2
K.MD.A.2  Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute, and describe the difference (e.g., directly compare the length of 10 cubes to a pencil and describe one as longer or shorter).	Unit K: Lessons 1, 2 Unit 1: Lesson 3

#### **KINDERGARTEN:** Geometry (G)

K.G.A Identify and describe shapes.	MPP units and lessons addressing standard
K.G.A.1  Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	Unit K: Lesson 4

#### **GRADE 1:** Operations and Algebraic Thinking (OA)

1.OA.A  Represent and solve problems involving addition and subtraction.	MPP units and lessons addressing standard
1.OA.A.1  Use addition and subtraction within 20 to solve word problems with unknowns in all positions (e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem).	Unit 1: Lessons 3, 4, 9



1.0A.C Add and subtract within 10.	MPP units and lessons addressing standard
1.0A.C.5  Relate counting to addition and subtraction (e.g., by using counting on 2 to add 2).	Unit K: Lessons 9, 10 Unit 1: Lessons 1, 2, 4
1.0A.C.6 Fluently add and subtract within 10.	Unit K: Lessons 9, 10 Unit 1: Lessons 1, 2

1.OA.D  Work with addition and subtraction equations.	MPP units and lessons addressing standard
1.0A.D.7  Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? $6 + 1 = 6 - 1$ , $7 = 8 - 1$ , $5 + 2 = 2 + 5$ , $4 + 1 = 5 + 2$ ).	Unit 1: Lesson 6
1.0A.D.8  Determine the unknown whole number in an addition or subtraction equation relating three whole numbers (e.g., determine the unknown number that makes the equation true in each of the equations $8 + \square = 11$ , $5 = \square - 3$ , $6 + 6 = \square$ ).	Unit 1: Lessons 6, 7, 8, 10 Unit 3: Lesson 7

#### **GRADE 1:** Number and Operations in Base Ten (NBT)

(Note: A range of algorithms may be used.)

1.NBT.B Understand place value.	MPP units and lessons addressing standard
1.NBT.B.2	Unit 1: Lessons 5, 7, 9, 10
Understand that the two digits of a two-digit number represent groups of tens and ones. Understand the following as special cases:	
a. 10 can be thought of as a group of ten ones—called a "ten."	
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	



1.NBT.C Use place value understanding and properties of operations to add and subtract.	MPP units and lessons addressing standard
1.NBT.C.4  Demonstrate understanding of addition within 100, connecting objects or drawings to strategies based on place value (including multiples of 10), properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form.	Unit 1: Lessons 7, 8 Unit 2: Lessons 3, 4 Unit 3: Lesson 7

## **GRADE 2:** Operations and Algebraic Thinking (OA)

2.OA.A  Represent and solve problems involving addition and subtraction.	MPP units and lessons addressing standard
2.OA.A.1  Use addition and subtraction within 100 to solve one- and two-step word problems. Represent a word problem as an equation with a symbol for the unknown.	Unit 2: Lesson 4

2.OA.B Add and subtract within 20.	MPP units and lessons addressing standard
2.OA.B.2  Fluently add and subtract within 20. By the end of Grade 2, know from memory all sums of two one-digit numbers.	Unit 2: Lessons 7



#### **GRADE 2:** Number and Operations in Base Ten (NBT)

(Note: A range of algorithms may be used.)

2.NBT.A Understand place value.	MPP units and lessons addressing standard
<ul> <li>2.NBT.A.1</li> <li>Understand that the three digits of a three-digit number represent groups of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones, and also equals 70 tens and 6 ones).</li> <li>Understand the following as special cases:</li> <li>a. 100 can be thought of as a group of ten tens—called a "hundred."</li> <li>b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</li> </ul>	Unit 2: Lessons 6, 8 Unit 3: Lessons 8, 9
2.NBT.A.2  Count within 1000; skip count by 5s, 10s and 100s.	Unit 2: Lessons 3, 6, 8 Unit 3: Lessons 2, 3, 4, 5, 6
2.NBT.A.3  Read and write numbers up to 1000 using base-ten numerals, number names, and expanded form.	Unit 2: Lessons 6, 8 Unit 3: Lessons 6, 8

2.NBT.B  Use place value understanding and properties of operations to add and subtract.	MPP units and lessons addressing standard
2.NBT.B.5  Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Unit 2: Lessons 3, 5, 8, 9, 10 Unit 3: Lesson 3
2.NBT.B.6 Add up to three two-digit numbers using strategies based on place value and properties of operations.	Unit 2: Lessons 3, 5, 10
2.NBT.B.7  Demonstrate understanding of addition and subtraction within 1000, connecting objects or drawings to strategies based on place value (including multiples of 10), properties of operations, and/ or the relationship between addition and subtraction. Relate the strategy to a written form.	Unit 2: Lessons 3, 4, 5, 6, 8, 9, 10 Unit 3: Lessons 3, 4, 5, 9



2.NBT.B  Use place value understanding and properties of operations to add and subtract.	MPP units and lessons addressing standard
2.NBT.B.8  Mentally add 10 or 100 to a given number in the range of 100 and 900, and mentally subtract 10 or 100 from a given number in the range of 100 and 900.	Unit 2: Lesson 6 Unit 3: Lesson 6
2.NBT.B.9  Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)	Unit 2: Lessons 3, 4, 5, 6, 8 Unit 3: Lessons 3, 4, 5, 6, 7, 8

#### **GRADE 2:** Measurement and Data (MD)

2.MD.B Relate addition and subtraction to length.	MPP units and lessons addressing standard
2.MD.B.6  Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,, and represent whole-number sums and differences within 100 on a number line diagram.	Unit 2: Lessons 1, 2, 3, 4 Unit 3: Lessons 3, 4, 5

#### **GRADE 3:** Operations and Algebraic Thinking (OA)

(Note: Grade 3 expectations in this domain are limited to whole number multiplication through 10 x 10 and whole number division with both quotients and divisors less than or equal to 10.)

3.OA.A  Represent and solve problems involving whole number multiplication and division.	MPP units and lessons addressing standard
3. OA.A.1 Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each).	Unit 3: Lessons 10, 11



3.0A.A  Represent and solve problems involving whole number multiplication and division.	MPP units and lessons addressing standard
3.0A.A.3  Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.	Unit 3: Lessons 10, 11  Note: These lessons do not deal with division.
3.0A.A.4  Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times 1 = 48$ , $5 = 1 \div 3$ , $6 \times 6 = 1$ .	Unit 3: Lesson 10

3.OA.B Understand properties of multiplication and the relationship between multiplication and division.	MPP units and lessons addressing standard
3.OA.B.5  Apply properties of operations as strategies to multiply and divide.  Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)	Unit 3: Lesson 11

3.OA.C Multiply and divide within 100.	MPP units and lessons addressing standard
3.OA.C.7  Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	Unit 3: Lessons 10, 11

3.OA.D  Solve problems involving the four operations, and identify and explain patterns in arithmetic.	MPP units and lessons addressing standard
3.0A.D.10  When solving problems, assess the reasonableness of answers using mental computation and estimation strategies, including rounding.	Unit 2: Lessons 1, 2 Unit 3: Lessons 1, 2



#### **GRADE 3:** Number and Operations in Base Ten (NBT)

(Note: A range of algorithms may be used.)

3.NBT.A  Use place value understanding and properties of operations to perform multi-digit arithmetic.	MPP units and lessons addressing standard
3.NBT.A.1  Use place value understanding to round whole numbers to the nearest 10 or 100.	Unit 3: Lessons 1, 2
3.NBT.A.2  Fluently add and subtract within 1000, using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Unit 2: Lesson 8 Unit 3: Lesson 6

#### **GRADE 3:** Number and Operations – Fractions (NF)

(Note: Grade 3 expectations are limited to fractions with denominators 2, 3, 4, 6, and 8.)

3.NF.A Understand fractions as numbers.	MPP units and lessons addressing standard
3.NF.A.3  Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	Unit 4: Lessons 1, 2, 6
a. Understand two fractions as equivalent if they have the same relative size compared to 1 whole.	
b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.	
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.	
<ul> <li>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols &gt;, =, or &lt;, and justify conclusions.</li> </ul>	



#### **GRADE 4:** Operations and Algebraic Thinking (OA)

4.0A.C Generate and analyze patterns.	MPP units and lessons addressing standard
4.0A.C.6  When solving problems, assess the reasonableness of answers using mental computation and estimation strategies, including rounding.	Unit 4: Lesson 11 Unit 5: Lesson 4

#### **GRADE 4:** Number and Operations in Base Ten (NBT)

(Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)

4.NBT.B  Use place value understanding and properties of operations to perform multi-digit arithmetic.	MPP units and lessons addressing standard
4.NBT.B.4  Fluently add and subtract multi-digit whole numbers using a standard algorithm.	Unit 2: Lessons 5, 9, 10 Unit 3: Lesson 9
4.NBT.B.5  Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Unit 3: Lesson 11



#### **GRADE 4:** Number and Operations—Fractions (NF)

(Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

4.NF.A Extend understanding of fraction equivalence and ordering.	MPP units and lessons addressing standard
4.NF.A.1  Explain why a fraction <i>alb</i> is equivalent to a fraction ( <i>n</i> x a)/( <i>n</i> x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.	Unit 4: Lessons 1, 2, 4, 6, 7, 8
4.NF.A.2  Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction).  a. Understand that comparisons are valid only when the two	Unit 4: Lessons 1, 4, 6
fractions refer to the same size whole.  a. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.	



4.NF.B  Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.	MPP units and lessons addressing standard
4.NF.B.3	4.NF.B.3.a
Understand a fraction $a/b$ with $a>1$ as a sum of unit fractions (1/b).	Unit 4: Lesson 9
a. Understand addition and subtraction of fractions as joining and	4.NF.B.3.b
separating parts referring to the same whole.	Unit 4: Lesson 3
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., $3/8 = 1/8 + 1/8 + 1/8$ ; $3/8 = 2/8 + 1/8$ ; $2 1/8 = 1 + 1 + 1/8 + $ or $2 1/8 = 8/8 + 8/8 + 1/8$ ).	4.NF.B.3.c Unit 4: Lesson 5
c. Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction).	
<ul> <li>Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.</li> </ul>	

4.NF.C Understand decimal notation for fractions, and compare decimal fractions.	MPP units and lessons addressing standard
4.NF.C.5  Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). For example, express 3/10 as 30/100, and 3/10 + 4/100 = 34/100. (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)	Unit 5: Lesson 1
4.NF.C.6  Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.	Unit 5: Lessons 1, 2, 4, 5 Unit 6: Lesson 4



#### **GRADE 5:** Number and Operations in Base Ten (NBT)

5.NBT.A Understand the place value system.	MPP units and lessons addressing standard
5.NBT.A.3	5.NBT.A.3.a
Read, write, and compare decimals to thousandths.	Unit 5: Lessons 3, 6
<ul> <li>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.</li> </ul>	5.NBT.A.3.b
b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Unit 5: Lesson 6

5.NBT.B  Perform operations with multi-digit whole numbers and with decimals to hundredths.	MPP units and lessons addressing standard
5.NBT.B.7  Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Unit 5: Lessons 7, 8, 9, 10, 11

5.NF.A Use equivalent fractions to add and subtract fractions.	MPP units and lessons addressing standard
5.NF.A.1  Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ ).	Unit 4: Lessons 7, 8
5.NF.A.2  Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g., recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).	Unit 4: Lesson 7



5.NF.B Use previous understandings of multiplication and division to multiply and divide fractions.	MPP units and lessons addressing standard
5.NF.B.4  Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction.	5.NF.B.4.b Unit 4: Lesson 10
<ul> <li>a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.</li> <li>b. Interpret the product of a fraction multiplied by a fraction (a/b)</li> </ul>	5.NF.B.4.a Unit 4: Lesson 11
x ( $c/d$ ). Use a visual fraction model and create a story context for this equation. For example, use a visual fraction model to show ( $2/3$ ) $x$ ( $4/5$ ) = $8/15$ , and create a story context for this equation. In general, ( $a/b$ ) $x$ ( $c/d$ ) = $ac/bd$ .	
c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	

#### **GRADE 6:** Ratio and Proportion (RP)

6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.	MPP units and lessons addressing standard
6.RP.A.1  Understand the concept of a ratio as comparing two quantities multiplicatively or joining/composing the two quantities in a way that preserves a multiplicative relationship. Use ratio language to describe a ratio relationship between two quantities. For example, "There were 2/3 as many men as women at the concert."	Unit 7: Lessons 1, 2, 8, 9, 10
6.RP.A.2  Understand the concept of a unit rate $a/b$ associated with a ratio $a:b$ with $b \neq 0$ , and use rate language (e.g., for every, for each, for each 1, per) in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)	Unit 7: Lessons 1, 5, 6



6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.	MPP units and lessons addressing standard
6.RP.A.3	6.RP.A.3
Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).	Unit 7: Lessons 1, 2, 7  6.RP.A.3.b
a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	Unit 7: Lessons 5, 6  6.RP.A.3.c  Unit 6: Lessons 1, 2, 6, 7, 9, 10
<ul> <li>Solve unit rate problems, including those involving unit pricing and constant speed.</li> </ul>	6.RP.A.3.d
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity). Solve percent problems with the unknown in all positions of the equation.	Unit 7: Lesson 11
<ul> <li>d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</li> </ul>	

## **GRADE 6:** The Number System (NS)

6.NS.B  Compute fluently with multi-digit numbers and find common factors and multiples.	MPP units and lessons addressing standard
6.NS.B.2 Fluently divide multi-digit numbers using a standard algorithm.	Unit 7: Lesson 9
6.NS.B.3  Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	Unit 5: Lesson 10 Unit 6: Lessons 6, 7, 11 Unit 7: Lesson 6



### **GRADE 6:** Expressions and Equations (EE)

6.EE.B  Reason about and solve one-variable equations and inequalities.	MPP units and lessons addressing standard
6.EE.B.5  Understand solving an equation or inequality as a process of reasoning to find the values of the variables that make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Unit 4: Lesson 11 Unit 6: Lessons 8, 9, 10 Unit 7: Lessons 8, 9, 10, 11
6.EE.B.6  Use variables to represent numbers and write expressions when solving mathematical problems and problems in real-world context; understand that a variable can represent an unknown number or any number in a specified set.	Unit 6: Lessons 8, 9, 10 Unit 7: Lessons 9, 10
6.EE.B.7  Solve mathematical problems and problems in real-world context by writing and solving equations of the form $x + p = q$ , $x - p = q$ , $px = q$ , and $x/p = q$ for cases in which $p$ , $q$ , and $x$ are all nonnegative rational numbers.	Unit 6: Lesson 9



#### **GRADE 7:** Ratio and Proportion (RP)

7.RP.A  Analyze proportional relationships and use them to solve mathematical problems and problems in real-world context.	MPP units and lessons addressing standard
7.RP.A.2	7.RP.A.2
Recognize and represent proportional relationships between quantities.	Unit 7: Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).	7.RP.A.2.a Unit 7: Lesson 2
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	7.RP.A.2.b Unit 7: Lesson 5
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.	
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	
7.RP.A.3	Unit 6: Lesson 11
Use proportional relationships to solve multi-step ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error).	



#### **GRADE 7:** The Number System (NS)

7.NS.A  Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers, except division by zero.	MPP units and lessons addressing standard
7.NS.A.2	7.NS.A.2.d
Multiply and divide integers and other rational numbers.	Unit 6: Lessons 3, 5
a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context.	
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real-world context.	
c. Apply properties of operations as strategies to multiply and divide rational numbers.	
d. Convert a rational number to decimal form using long division; know that the decimal form of a rational number terminates in Os or eventually repeats.	



## **Math Pathways & Pitfalls Lessons**

# Aligned With Arizona Mathematics Standards (AMS)

Unit K: Early Math & Number Concepts	Arizona Mathematics Standards
Lesson 1: Which Train Is Longer?	K.CC.C.6 K.MD.A.1 K.MD.A.2
Lesson 2: Which Train Is Shorter?	K.CC.C.6 K.MD.A.1 K.MD.A.2
Lesson 3: Same Number or Not?	K.CC.B.4.a K.CC.B.4.b K.CC.B.5 K.CC.C.6
Lesson 4: Which Row Has More?	K.CC.B.4.a K.CC.B.4.b K.CC.B.4.c K.CC.B.5 K.CC.C.6 K.G.A.1
Lesson 5: Which Row Has Fewer?	K.CC.B.4.a K.CC.B.4.b K.CC.B.4.c K.CC.B.5 K.CC.C.6
Lesson 6: Counting Ladybug Dots	K.CC.A.3 K.CC.B.4.a K.CC.B.4.b K.CC.B.5
Lesson 7: Counting Beads	K.CC.A.3 K.CC.B.4.a K.CC.B.4.b K.CC.B.5



Unit K: Early Math & Number Concepts	Arizona Mathematics Standards
Lesson 8: Count Out 3	K.CC.A.3 K.CC.B.4.a K.CC.B.4.b K.CC.B.5 K.OA.A.1
Lesson 9: Cross Off 4	K.CC.B.5 K.OA.A.1 K.OA.A.2 1.OA.C.5 1.OA.C.6
Lesson 10: Counting Extras	K.CC.B.5 K.CC.C.6 K.OA.A.1 K.OA.A.2 1.OA.C.5

Unit 1: Number, Place Value, & Operations	Arizona Mathematics Standards
Lesson 1: Points on a Number Line	K.CC.A.2 1.OA.C.5 1.OA.C.6
Lesson 2: Counting On a Few	K.CC.A.2 1.OA.C.5 1.OA.C.6
Lesson 3: How Many More?	K.CC.B.4.c K.CC.C.6 K.OA.A.2 K.MD.A.2 1.OA.A.1
Lesson 4: Addition Story Problems	K.CC.A.2 1.OA.A.1 1.OA.C.5 1.OA.C.6
Lesson 5: Tens and Ones Flip	1.NBT.B.2
Lesson 6: What Number Is Hiding? (Algebra Readiness)	1.OA.D.7 1.OA.D.8



Unit 1: Number, Place Value, & Operations	Arizona Mathematics Standards
Lesson 7: Adding to Teen Numbers	K.NBT.A.1 1.OA.D.8 1.NBT.B.2 1.NBT.C.4
Lesson 8: Adding Tens and Ones	1.OA.D.8 1.NBT.C.4
Lesson 9: How Many Are Left?	1.OA.A.1 1.NBT.B.2
Lesson 10: Teen Number Subtraction	1.OA.C.6 1.OA.D.8 1.NBT.B.2

Unit 2: Number, Place Value, & Operations	Arizona Mathematics Standards
Lesson 1: Number Line Points	2.MD.B.6 3.OA.D.10
Lesson 2: Marking Points on a Number Line	2.MD.B.6 3.OA.D.10
Lesson 3: Add a Few	1.NBT.C.4 2.NBT.A.2 2.NBT.B.5 2.NBT.B.6 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 4: What's the Difference?	1.0A.C.6 1.NBT.C.4 2.0A.A.1 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 5: Don't Squeeze the Digits	2.NBT.B.5 2.NBT.B.6 2.NBT.B.7 2.NBT.B.9 4.NBT.B.4



Unit 2: Number, Place Value, & Operations	Arizona Mathematics Standards
Lesson 6: Adding On More Tens or Ones	2.NBT.A.1 2.NBT.A.2 2.NBT.A.3 2.NBT.B.7 2.NBT.B.8 2.NBT.B.9
Lesson 7: What Goes in the Blank? (Algebra Readiness)	2.OA.B.2
Lesson 8: Place Value Hints (Algebra Readiness)	2.NBT.A.1 2.NBT.A.2 2.NBT.A.3 2.NBT.B.5 2.NBT.B.7 2.NBT.B.9 3.NBT.A.2
Lesson 9: Minus a Few	2.NBT.B.5 2.NBT.B.7 4.NBT.B.4
Lesson 10: Regroup and Subtract	2.NBT.B.5 2.NBT.B.6 2.NBT.B.7 4.NBT.B.4

Unit 3: Number, Regrouping, & Operations	Arizona Mathematics Standard
Lesson 1: Number Line Sense	3.NBT.A.1 3.OA.D.10
Lesson 2: Marking Points for Numbers	2.NBT.A.2 3.NBT.A.1 3.OA.D.10
Lesson 3: Add On a Bit More	2.NBT.A.2 2.NBT.B.5 2.NBT.B.7 2.NBT.B.9 2.MD.B.6



Unit 3: Number, Regrouping, & Operations	Arizona Mathematics Standard
Lesson 4: A Little Less	2.NBT.A.2 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 5: Finding the Difference	2.NBT.A.2 2.NBT.B.7 2.NBT.B.9 2.MD.B.6
Lesson 6: Add On Using Place Value	2.NBT.A.2 2.NBT.A.3 2.NBT.B.8 2.NBT.B.9 3.NBT.A.2
Lesson 7: What Number Is Missing? (Algebra Readiness)	1.OA.D.8 1.NBT.C.4 2.NBT.B.9
Lesson 8: Values of Digits (Algebra Readiness)	2.NBT.A.1 2.NBT.A.3 2.NBT.B.9
Lesson 9: Regroup a Ten	2.NBT.A.1 2.NBT.B.7 4.NBT.B.4
Lesson 10: When Do You Multiply?	3.OA.A.1 3.OA.A.3 (Note: This lesson does not incorporate division.) 3.OA.A.4 3.OA.C.7
Lesson 11: Making Sense and Multiplying	3.OA.A.1 3.OA.A.3 (Note: This lesson does not incorporate division.) 3.OA.B.5 3.OA.C.7 4.NBT.B.5



Unit 4: Fractions	Arizona Mathematics Standard
Lesson 1: Naming Equivalent Fractions	3.NF.A.3 4.NF.A.1 4.NF.A.2
Lesson 2: Fraction Flags	3.NF.A.3 4.NF.A.1
Lesson 3: Fractions on a Number Line	4.NF.B.3.b
Lesson 4: Naming Fractions in Lowest Terms	4.NF.A.1 4.NF.A.2
Lesson 5: Equal Fractions and Mixed Numbers	Supports cluster 4.NF.B.3
Lesson 6: Comparing Fraction Amounts	3.NF.A.3 4.NF.A.1 4.NF.A.2
Lesson 7: Adding Fractions	4.NF.A.1 5.NF.A.1 5.NF.A.2
Lesson 8: Sums More or Less Than 1	4.NF.A.1 5.NF.A.1
Lesson 9: Fraction Subtraction	4.NF.B.3.a
Lesson 10: Multiplying Fractions	5.NF.B.4
Lesson 11: Unknown Fractional Parts of a Number (Algebra Readiness)	4.OA.C.6 5.NF.B.4.a 6.EE.B.5



Unit 5: Decimals	Arizona Mathematics Standards
Lesson 1: Shading Decimal Amounts	4.NF.C.5 4.NF.C.6
Lesson 2: Decimals Are Fractions Too	4.NF.C.6
Lesson 3: Naming and Drawing Decimal Amounts	5.NBT.A.3.a
Lesson 4: Approximating Decimals for Fractions	4.0A.C.6
Lesson 5: Decimals More or Less Than 1	4.NF.C.6 4.OA.C.6
Lesson 6: Comparing and Ordering Decimals	5.NBT.A.3.a 5.NBT.A.3.b
Lesson 7: Finding Sums and Differences Mentally	5.NBT.B.7
Lesson 8: Compute Decimal Sums and Differences	5.NBT.B.7
Lesson 9: Multiplying Decimals	5.NBT.B.7
Lesson 10: Dividing Decimals	5.NBT.B.7 6.NS.B.3
Lesson 11: Unknown Decimal Addends (Algebra Readiness)	5.NBT.B.7

Unit 6: Percents	Arizona Mathematics Standards
Lesson 1: Percent Names for Shaded Areas	6.RP.A.3.c
Lesson 2: Relating Percent and Fraction Amounts	6.RP.A.3.c
Lesson 3: Fractions Into Percents	7.NS.A.2.d
Lesson 4: Percents, Fractions, and Decimals	4.NF.C.6
Lesson 5: Changing Ratios to Percents	7.NS.A.2.d
Lesson 6: Finding Percent Mentally	6.RP.A.3.c 6.NS.B.3
Lesson 7: More or Less Than 10%	6.RP.A.3.c 6.NS.B.3



Unit 6: Percents	Arizona Mathematics Standards
Lesson 8: Translate Into Percent Equations (Algebra Readiness)	6.EE.B.5 6.EE.B.6
Lesson 9: Percent Equations (Algebra Readiness)	6.RP.A.3.c 6.EE.B.5 6.EE.B.6 6.EE.B.7
Lesson 10: Percent Problems and Proportions (Algebra Readiness)	6.RP.A.3.c 6.EE.B.5 6.EE.B.6
Lesson 11: Percent Discount or Markup (Algebra Readiness)	6.NS.B.3 7.RP.A.3

Unit 7: Ratios & Proportions	Arizona Mathematics Standards
Lesson 1: Are the Ratios Equal?	6.RP.A.1 6.RP.A.2 6.RP.A.3 7.RP.A.2
Lesson 2: Making Equal Ratios	6.RP.A.1 6.RP.A.3 7.RP.A.2 7.RP.A.2.a
Lesson 3: 25 Times as Many	7.RP.A.2
Lesson 4: 2.5 Times as Many Jumps	7.RP.A.2
Lesson 5: How Many for 1?	6.RP.A.2 6.RP.A.3.b 7.RP.A.2 7.RP.A.2.b
Lesson 6: Comparing Ratios	6.RP.A.2 6.RP.A.3.b 6.NS.B.3 7.RP.A.2
Lesson 7: Simplify to Find Equivalent Ratios	6.RP.A.3 7.RP.A.2



Unit 7: Ratios & Proportions	Arizona Mathematics Standards
Lesson 8: Solve for x in Proportions (Algebra Readiness)	6.RP.A.1 6.EE.B.5 7.RP.A.2
Lesson 9: Setting Up Proportions (Algebra Readiness)	6.RP.A.1 6.NS.B.2 6.EE.B.5 6.EE.B.6 7.RP.A.2
Lesson 10: Similar Figures (Algebra Readiness)	6.RP.A.1 6.EE.B.5 6.EE.B.6 7.RP.A.2
Lesson 11: Proportion Problem or Not? (Algebra Readiness)	6.RP.A.3.d 6.EE.B.5

### References

Council of Chief State School Officers. (2017). *Common Core State Standards for Mathematics*. https://ccsso.org/sites/default/files/2017-10/MathStandards50805232017.pdf

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*.

National Council of Teachers of Mathematics. (2017a). *Taking action: Implementing effective mathematics teaching practices in grades 6–8*.

National Council of Teachers of Mathematics. (2017b). *Taking action: Implementing effective mathematics teaching practices in K—grade 5*.

National Council of Teachers of Mathematics. (2020a). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations*.

National Council of Teachers of Mathematics. (2020b). *Catalyzing change in middle school mathematics: Initiating critical conversations*.